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Problem- Spring constant of a rope

A dynamic rope is used for belaying a climber. This rope has a moderate elasticity which allows it to arrest a climber fall without large shock loading thus minimizing the force on the climber. Rope manufacturers specify the elasticity of a climb rope by the fractional static elongation s defined by $s = x/L$ where x is the change in length of a rope when a standard mass m_s of 80 kg is hung on it and L is the original length of the rope. Note that this is sometimes specified as a percentage so that the percent static elongation is $s = 100\% \times x/L$. Assuming the rope obeys Hooke's law, a spring constant k can be defined for it using $F = kx$. We will assume the fractional static elongation is 0.07 and the rope length is 20 m.

a) Determine the spring constant for this dynamic rope if its length is 20 m.

b) How does the spring constant k scale with the length of the rope. What happens if we reduce the rope length by a factor of 2?

c) If the climber takes a fall on this rope and bounces, estimate the time for one bounce. Assume climber has some mass $m_c = 60$ kg.

[solution is on next page]

Solution- Spring constant of a rope

A dynamic rope is used for belaying a climber. This rope has a moderate elasticity which allows it to arrest a climber fall without large shock loading thus minimizing the force on the climber. Rope manufacturers specify the elasticity of a climb rope by the fractional static elongation s defined by $s = x/L$ where x is the change in length of a rope when a standard mass m_s of 80 kg is hung on it and L is the original length of the rope. Note that this is sometimes specified as a percentage so that the percent static elongation is $s = 100\% \times x/L$. Assuming the rope obeys Hooke's law, a spring constant k can be defined for it using $F = kx$. We will assume the fractional static elongation is 0.07 and the rope length is 20 m.

a) Determine the spring constant for this dynamic rope if its length is 20 m.

> The mass hung on the end of the rope m creates a force mg producing the extension of the rope. The extension of the rope can also be written in terms of s:
>
> $s = x/\mathrm{L}$ so that $x = sL$
>
> Then using the definition of k:
>
> $F = kx \;\Rightarrow\; k = F/x = mg/sL = 560$ N/m   (s must be a fraction, not a %)

b) How does the spring constant k scale with the length of the rope. What happens if we reduce the rope length by a factor of 2?

> According to part a), k scales as $1/L$ so if we reduce L by a factor of 2 then k doubles.
>
> This makes sense. Reducing the length of a climbing rope gives you a less springy rope and this is reflected in the fact that k which measures stiffness is increased.

c) If the climber takes a fall on this rope and bounces, estimate the time for one bounce. Assume climber has some mass $m_c = 60$ kg.

> This system (climber bouncing on a rope) is a harmonic oscillator (albeit heavily damped).
>
> The angular frequency, $\omega = \sqrt{(k/m)}$ and the period (time for one bounce) is $T = 2\pi/\omega$. *So*
>
> $T = 2\pi/\sqrt{(k/m)} =$ 2.1 seconds.