

**Problem - Finding the average force on landing.**

Consider a climber of mass  $m$  on a climb of height  $H$ . Assuming there is no slack in the rope and the belay device is at ground level, the amount of rope between the belayer and the climber is  $2H - h$ . Define:  $q \equiv mg / \kappa$  where  $\kappa$  is the spring constant per unit length of rope.

a) Show that the spring constant for the rope is

$$k = \frac{\kappa}{(2H - h)}.$$

b) Find an expression for the speed at which the climber hits the ground should they slip. Use this result to show that if they slip they will hit the ground only if  $h < 4Hq / (1 + 2q)$ .

Hint: use conservation of energy.

c) The average value of the tension in the belay rope during landing may be written:

$$T_{avg} \approx (T_A + T_B) / 2 = k(h + d / 2).$$

Model the landing of the climber by assuming their center of mass drops a distance  $d$  during landing. Prove that the average value of the force exerted by ground on climber during landing is:

$$F = mg \left[ 1 + \frac{h}{d} - \frac{(h + d)^2}{2dq(2H - h)} \right]$$

Hint: Use Newton's 2<sup>nd</sup> law.

d) Find the value of  $h$  when this force is a maximum.

e) Prove that this maximum force is

$$F_{max} = mg \left( 1 + \frac{1}{q} (1 - \sqrt{1 + 2q}) \right) \left( 1 + \frac{2H}{d} \right)$$

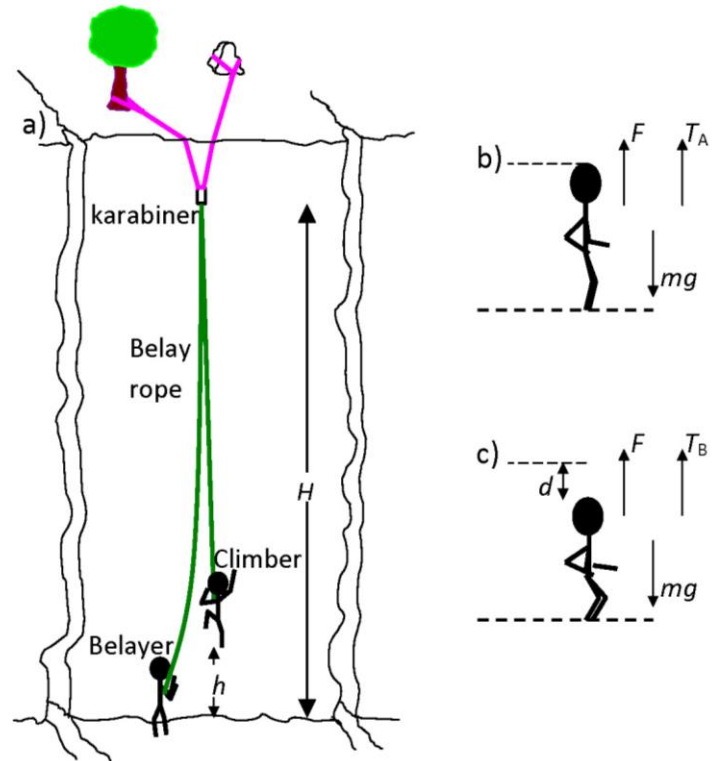


Figure 1. a) A top roped climber ascends a climb of height  $H$ . A belay rope passes through a double karabiner at the top of the climb and connects the climber to a belayer who controls the rope via a friction device. The climber slips and b) initially touches the ground as the belay rope stretches and c) comes to rest. During the landing the climber lowers their center of mass an amount  $d$  by bending their knees. The forces on the climber are shown to the right.