Backpack Problem

To create a model that will help determine the maximum backpack weight a hiker can carry, consider a human frame that can be scaled up in size (all three dimensions are scaled equally). We assume that some part of the human frame, e.g. the muscles in the thigh, limit the amount of weight the person can carry. The weight borne by 'limiting body part' we call the carrying capacity and is $W_C = \alpha W_h + W_{bp}$ where W_h and W_{bp} are the of the hiker weight and the maximum backpack respectively. The quantity α is the fraction of the body weight supported by the limiting body structure.

a) Give an argument that

$$W_{bp} = \left(\alpha W_{h0} + W_{bp0}\right)\left(\frac{W_h}{W_{h0}}\right)^{2/3} - \alpha W_h,$$

where W_{h0} and W_{bp0} are known values of the hiker weight and the maximum backpack weight for a particular hiker.

b) Differentiate W_{bp} with respect to W_h to show that:

$$W_h|_{(W_{bp}=W_{bp,max})} = \frac{8}{27\alpha}\left(\alpha W_{h0} + W_{bp0}\right)^3\left(\frac{1}{\alpha W_{h0}}\right)^2.$$

Be sure to give an argument that this is a maximum (not a minimum or inflection point)

c) Show that the value of the maximum backpack weight is:

$$W_{\text{bp,max}} = \frac{4}{27}\left(\alpha W_{h0} + W_{bp0}\right)^3\left(\frac{1}{\alpha W_{h0}}\right)^2.$$

d) Consider the case where W_{h0} and W_{bp0} are 700 N and 300 N respectively and α is 0.75 corresponding to somewhere in the lower back. Find the values of $W_h|_{(W_{bp}=W_{bp,max})}$ and $W_{\text{bp,max}}$.

e) Briefly discuss possible limitations of this model – there are several.

This model will give at best a very approximate estimate of maximum backpack weight for a person. The most significant contribution of this model to backpacking it that this model nicely points out that the backpack weight you carry *should not* be proportional to your actual weight!