

A Brief Introduction to Magnetism

M. J. O'Shea

Kansas State University

mjoshea@phys.ksu.edu

Periodic Table

H¹ 4K hcp 3.75 6.12		High T_c, low anisotropy																He⁴ 2K hcp 3.57 5.83					
Li 78K bcc 3.491	Be hcp 2.27 3.59																	B rhomb. 4.05	C diamond 3.567	N 20K cubic 5.66 (N ₂)	O complex (O ₂)	F	Ne fcc 4.46
Na 5K bcc 4.225	Mg hcp 3.21 5.21	Crystal structure, a lattice parameter, in Å c lattice parameter, in Å																Al fcc 4.05	Si diamond 5.430	P complex	S complex	Cl complex (Cl ₂)	Ar 4K fcc 5.31
K 5K bcc 5.225	Ca fcc 5.58	Sc hcp 3.31 5.27	Ti hcp 2.95 4.68	V bcc 3.03	Cr bcc 2.88	Mn cubic complex	Fe bcc 2.87	Co hcp 2.51 4.07	Ni fcc 3.52	Cu fcc 3.61	Zn hcp 2.66 4.95	Ga complex	Ge diamond 5.658	As rhomb.	Se hex. chains	Br complex (Br ₂)	Kr 4K fcc 5.64						
Rb 5K bcc 5.585	Sr fcc 6.08	Y hcp 3.65 5.73	Zr hcp 3.23 5.15	Nb bcc 3.30	Mo bcc 3.15	Tc hcp 2.74 4.40	Ru hcp 2.71 4.28	Rh fcc 3.80	Pd fcc 3.89	Ag fcc 4.09	Cd hcp 2.98 5.62	In tetr. 3.25 4.95	Sn (α) diamond 6.49	Sb rhomb.	Te hex. chains	I complex (I ₂)	Xe 4K fcc 6.13						
Cs 5K bcc 6.045	Ba bcc 5.02	La hex. 3.77 ABAC	Hf hcp 3.19 5.05	Ta bcc 3.30	W bcc 3.16	Re hcp 2.76 4.46	Os hcp 2.74 4.32	Ir fcc 3.84	Pt fcc 3.92	Au fcc 4.08	Hg rhomb.	Tl hcp 3.46 5.52	Pb fcc 4.95	Bi rhomb.	Po sc 3.34	At —	Rn —						
Fr —	Ra —	Ac fcc 5.31																					
			Ce fcc 5.16	Pr hex. 3.67 ABAC	Nd hex. 3.66	Pm —	Sm complex	Eu bcc 4.58	Gd hcp 3.63 5.78	Tb hcp 3.60 5.70	Dy hcp 3.59 5.65	Ho hcp 3.58 5.62	Er hcp 3.56 5.59	Tm hcp 3.54 5.56	Yb fcc 5.48	Lu hcp 3.50 5.55							
			Th fcc 5.08	Pa tetr. 3.92 3.24	U complex	Np complex	Pu complex	Am hex. 3.64 ABAC	Cm —	Bk —	Cf —	Es —	Fm —	Md —	No —	Lr —							

High T_c,
low anisotropy

Crystal structure.
a lattice parameter, in Å
c lattice parameter, in Å

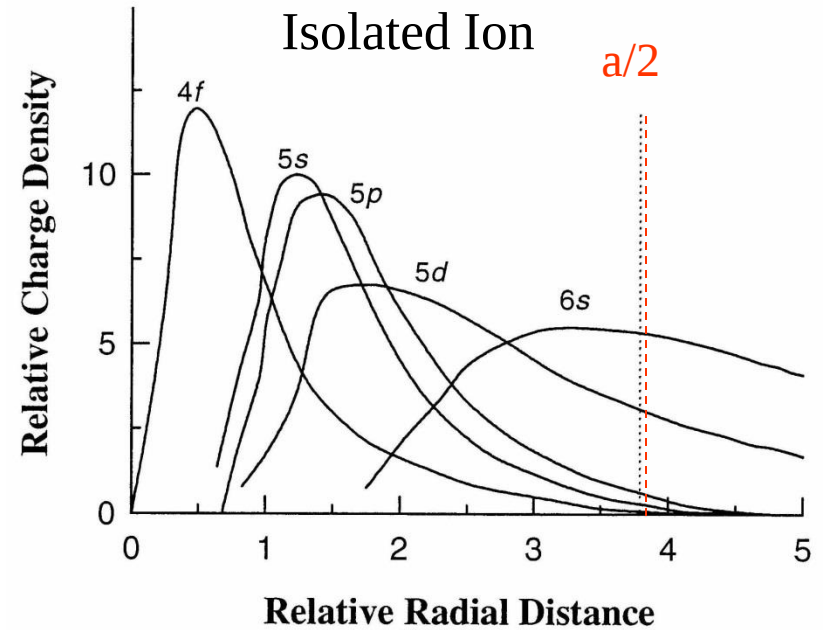
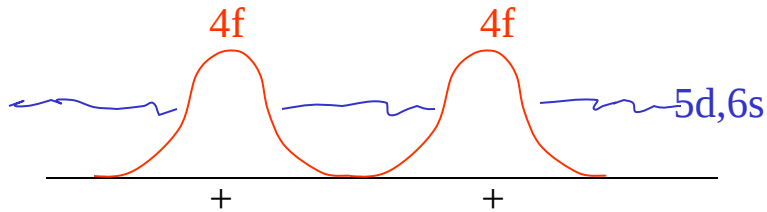
Low T_c,
Some have high anisotropy

Rare-earth and transition metals

Rare-earth metals:

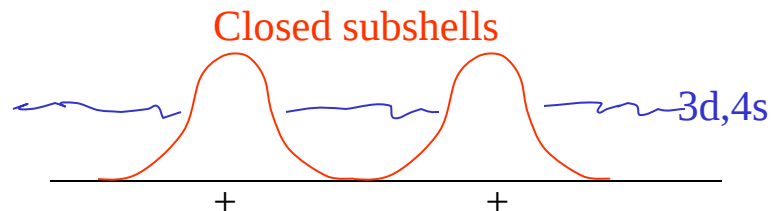
localized partially filled 4f shell,
5d,6s bands

Magnetic moment



Transition metals: localized filled subshells,
3d,4s bands

Magnetic moment



Magnetic Moment – rare-earth

Rare-Earth metals (Gd, Tb, Dy, Nd, Sm ...)

- magnetic moment resides on 4f electrons
- neighboring 4f-shells do not overlap, are atomic like.
- Hund's rules * orbital (L) and spin (S) angular momentum
- $\mu = \mu(L, S)$

Assume rare-earth is trivalent (usual case in metals)

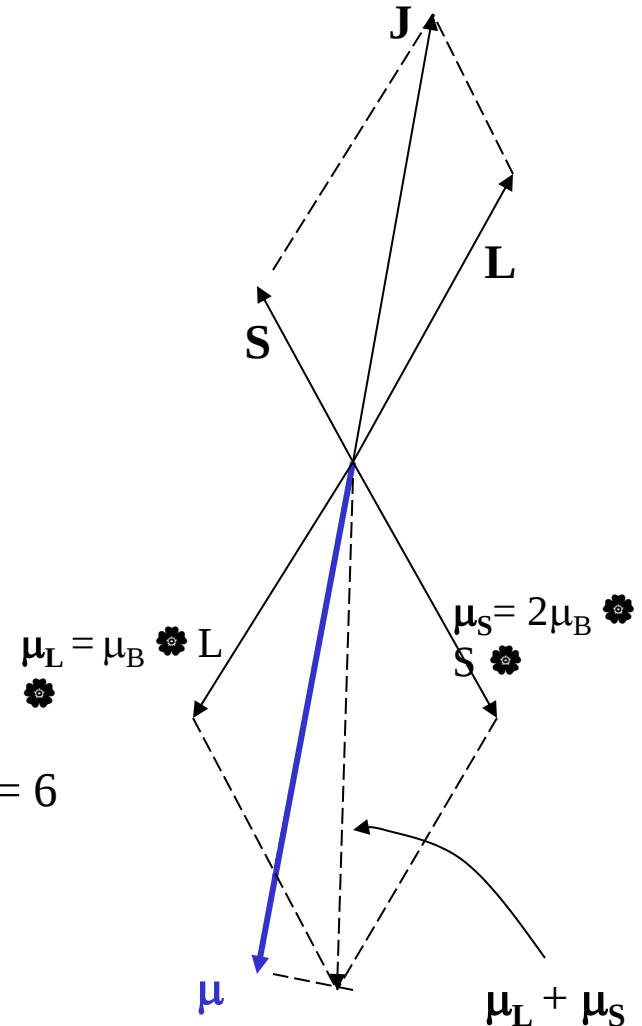
Use Hund's rules to find L, S, J.

- maximize S ($= \sum m_s$)
- maximize L ($= \sum m_l$)
- $J = *L - S*$, shell less than half full,
 $J = *L + S*$, shell more than half full

e.g. Tb: $[\text{Xe}]4f^8$ $5d^2 6s^1$ $S = 3, L = 3, J = *L + S* = 6$
↑ ↑
Tb³⁺ conduction

$$\mu = \mu(L, S) = g \mu_B J = 9\mu_B$$

$$\left[g = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} \right]$$



Origin of Magnetic Ordering

$$H = -\hbar^2/2m \nabla^2 \Psi + U(r)\Psi + (\text{e-e interaction term}) + (\text{dipole interaction term}) + \dots$$

Dipolar term is only explicit spin-spin interaction term ✱ small
Cannot account for magnetic ordering

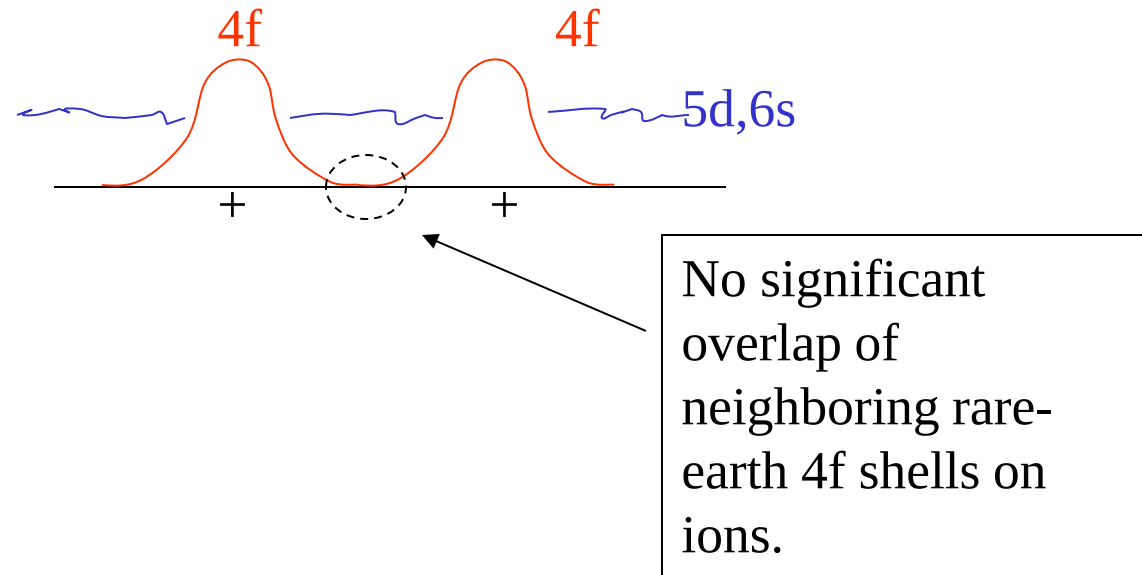
$$E_{\text{dipole}}/k_B \approx 1 \text{ K}$$

Pauli exclusion principle ✱ e-e interaction between parallel spin (✱✱) electrons is weaker than e-e interaction between antiparallel spin (✱✱) electrons.

$$\text{Modeled as } -JS_i \cdot S_j$$

Magnetic Ordering – rare-earths

Magnetic moment on one 4f shell polarizes the conduction (5d,6s) electrons and a neighboring 4f ion feels this polarization.



This weak indirect interaction leads to magnetic ordering below room temperature for the rare-earths.

Resulting order can be complex:

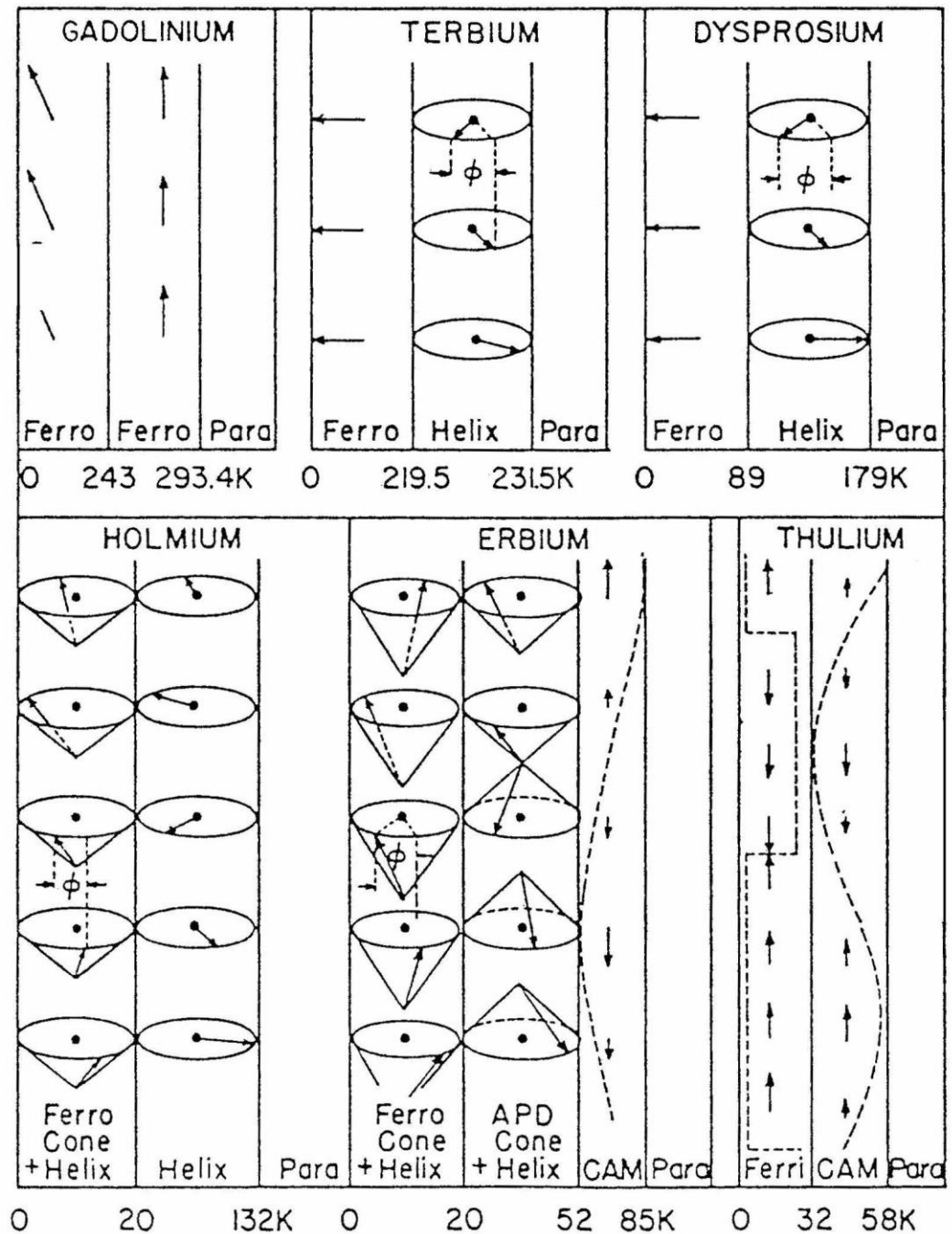
Gd, Tb, Dy are all ferromagnetic at low enough temperatures.

At higher temperatures Tb, Dy (< 300 K) they have a non-collinear magnetic order.

Elemental Rare-Earths

Magnetic ordering
determined from
neutron diffraction

[Wohlfarth,
'Ferromagnetic Materials']

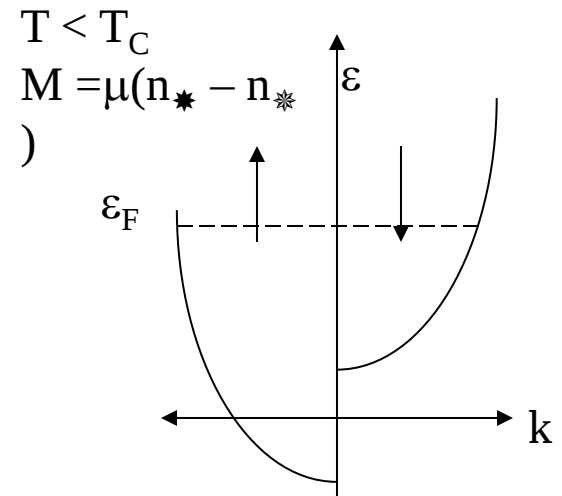
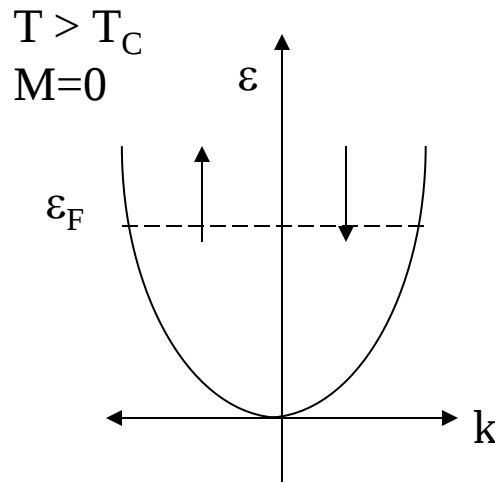


Magnetic Moment and T_c – transition metals

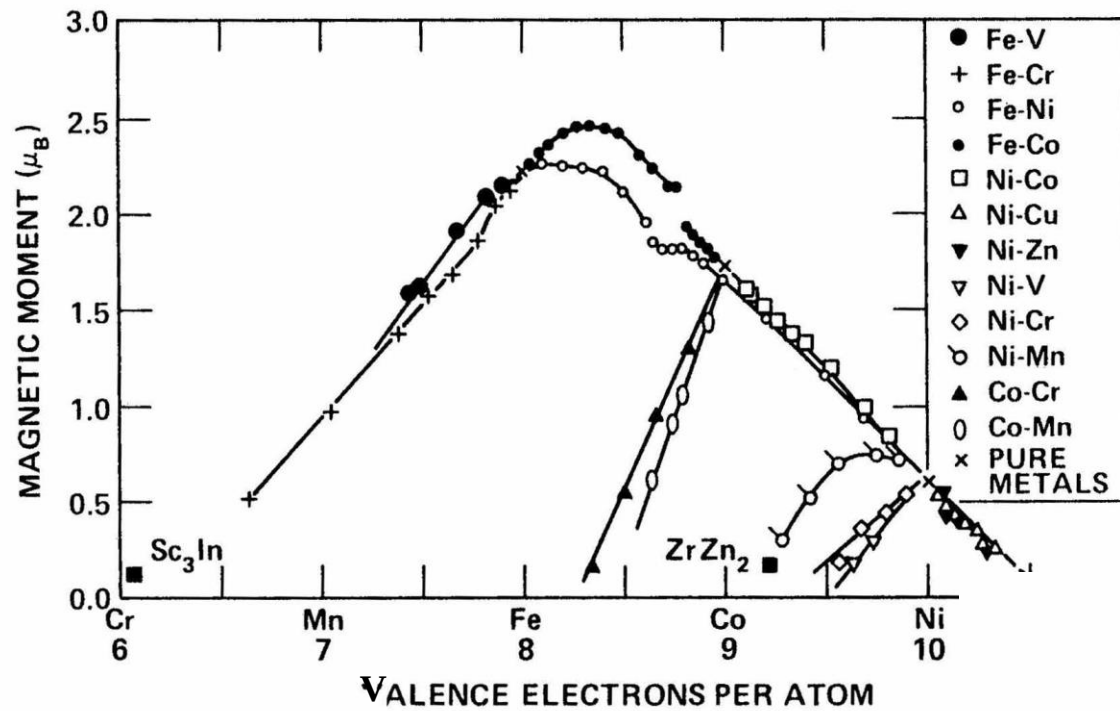
Transition metals (Fe, Co, Ni)

- magnetic moment resides on d electrons
- neighboring d-shells overlap and broaden into a d-band.
- orbital (L) angular momentum is mostly quenched
- $\mu = \mu(S)$

Model: $-J\mathbf{S}_i \cdot \mathbf{S}_j$.
Ferromagnetic if $J > 0$.

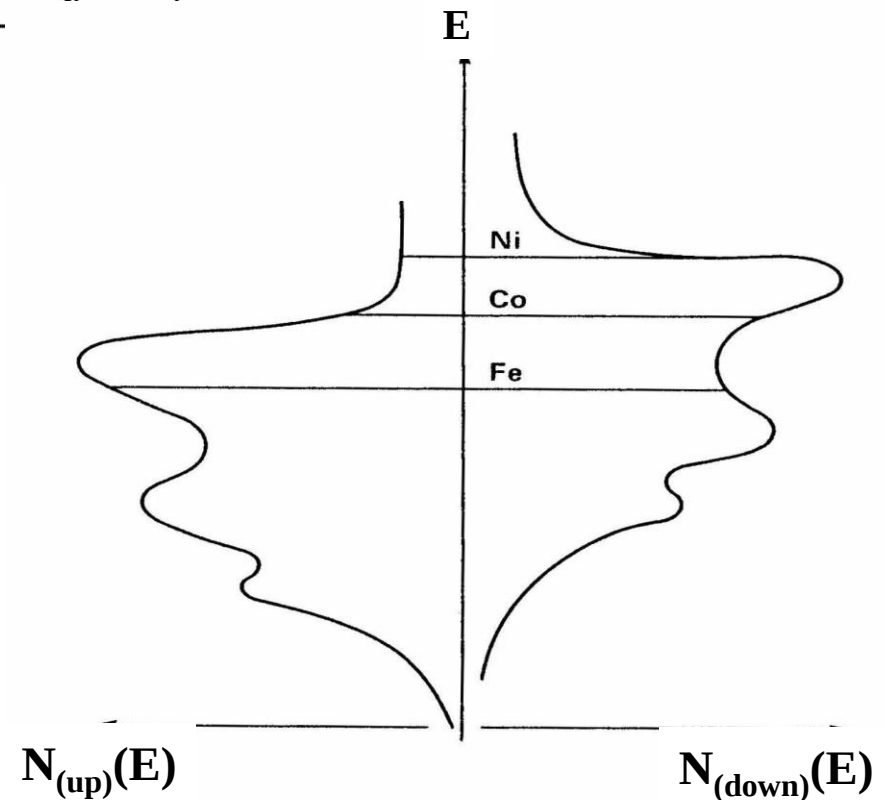


Rigid Band model – Transition metals



See Kittel, 'Introduction to
Solid State Physics'

Note: rigid band model not correct, but
does give correct qualitative picture



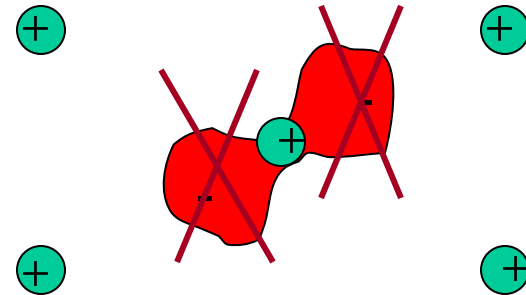
Magnetic Anisotropy

Origin of Magnetic anisotropy

1) Interaction between a local quadrupole moment (L) and electric field gradients.

2) $\mu = \mu(L, S)$

} $\Rightarrow$ Magnetic anisotropy



Strength of Magnetic anisotropy

Rare-Earths $\star$ large L $\star$ strong magnetocrystalline anisotropy

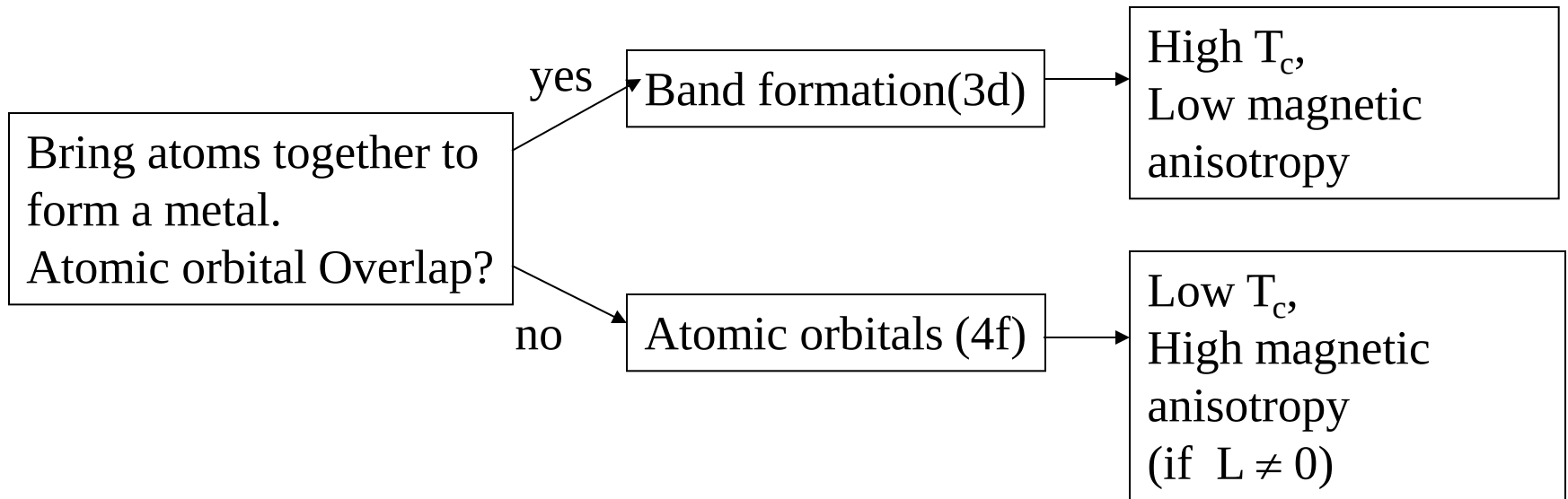
Transition metals $\star$ small L $\star$ weak magnetocrystalline anisotropy

Metal	K_1 (J/m ³)
Co	7×10^5
Tb	-5.6×10^7
Dy	-5.5×10^7
Gd	-1.2×10^5

} All hexagonal : $E_A = K_1 \sin^2\theta + K_2 \sin^4\theta \dots\dots\dots$

Note that for Fe, Ni: $E_A =$ (expression with cubic symmetry)

Magnetic anisotropy and ordering temperature

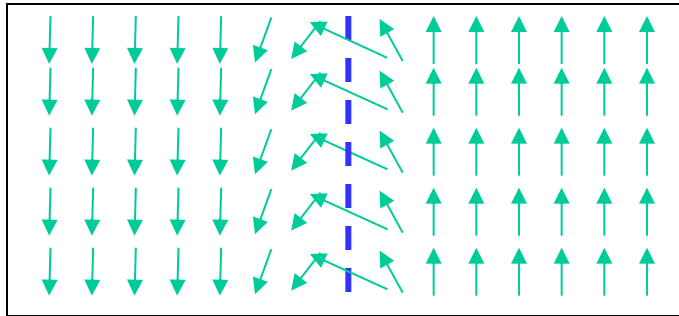


Magnetic elements

	Rare-Earth (Nd, Gd, Dy, Tb)	Transition metal (Fe, Co, Ni)
Moment	$\mu = (\mu_S + \mu_L)_J$, e.g. Tb $9.5\mu_B$ Gd $8.0\mu_B$ Nd $3.5\mu_B$ (f electrons)	$\mu = \mu_S + \Delta(\mu_L)$ e.g. Fe $2.2\mu_B$ Co $1.7\mu_B$ (d electrons)
Magnetic Ordering, T_c , and exchange	Indirect exchange, weak T_C : Tb 220 K Gd 293 K Nd 20 K	Direct exchange, strong T_C : Fe 1043 K Co 1388 K
Magnetic Anisotropy-	Unquenched L. Large anisotropy K: Dy $\sim -5.6 \times 10^7 \text{ J/m}^3$ Tb $\sim -5.5 \times 10^7$ (Gd $\sim 1 \times 10^6$)	Quenched L. Small anisotropy K: Fe $\sim 5 \times 10^4 \text{ J/m}^3$ Co $\sim 4 \times 10^5$

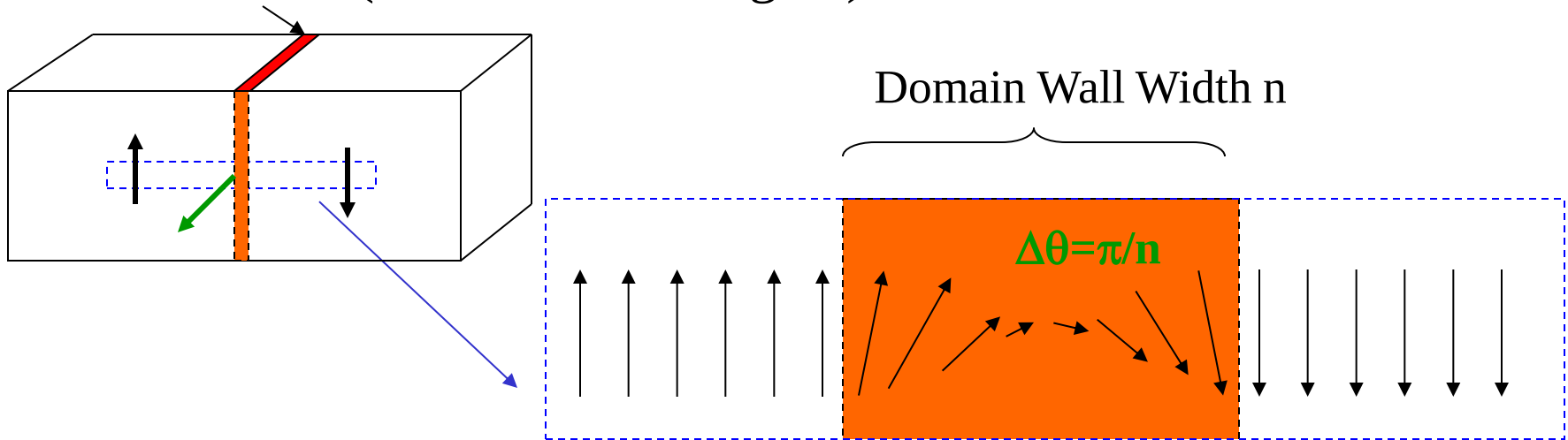
Why do domains form?

$$\Delta F = + E(\text{domain wall}) - \text{dipolar energy}$$



domain wall, area $A \sim V^{2/3}$

Domain Walls (uniaxial ferromagnet)



$$\Delta E_{\text{ex}} = n [(-Js^2 \cos \pi/n) - (-Js^2)] = Js^2 \pi^2/n,$$

$$\Delta E_{\text{anis}} = n (K/2)$$

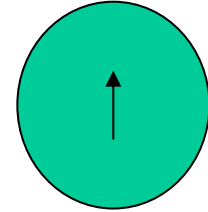
Minimize wrt n : $n(\text{equil}) = \sqrt{(2Js^2 \pi^2/K)} \rightarrow \text{domain wall width}$

and $\sigma_d = (s\pi/a^2) \sqrt{(2JK)} \rightarrow \text{domain wall energy/area}$

- anisotropic materials have narrow walls, Tb 2 nm
- isotropic materials have thicker walls, Fe 30 nm
- (Block wall for bulk and thick films)
- (Neel walls for thin films)

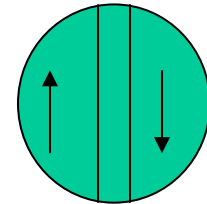
Single Domains

To estimate the size of a single magnetic domain consider a sphere, radius r :



$$\begin{aligned}\text{Magnetostatic energy (single domain)} &= -H_D \cdot M_t \\ &= \frac{1}{3} \mu_0 M_s^2 \left(\frac{4}{3} \pi r^3 \right)\end{aligned}$$

Domain wall energy = $\sigma \pi r^2$ (two domains)



When these two energies are equal, $r = r_{sd}$ giving,

$$r_{sd} = 9(AK)^{1/2} / \mu_0 M_s^2$$

(Note: this is a rough estimate, ignored other possibilities such as curling mode)

For a spherical Fe particle: $r_{sd} = 25$ nm.

For $r < r_{sd}$, magnetization reversal cannot occur via domain wall movement

How does r_{sd} depend on shape?

How does r_{sd} depend on anisotropy?

Superparamagnetism

Flip rate due to thermal fluctuations:

$$1/\tau = \nu \exp(-\Delta E/k_B T)$$

$$2.4 \times 10^9 \text{ s}^{-1}$$

Depends on size of particle
(=KV)

size (nm)	τ
10	1 week
20	10^{104} years

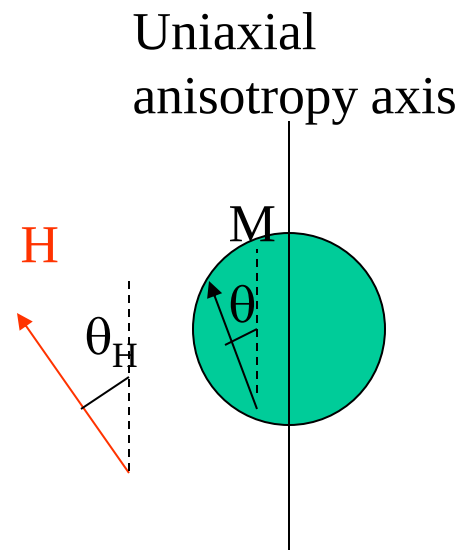
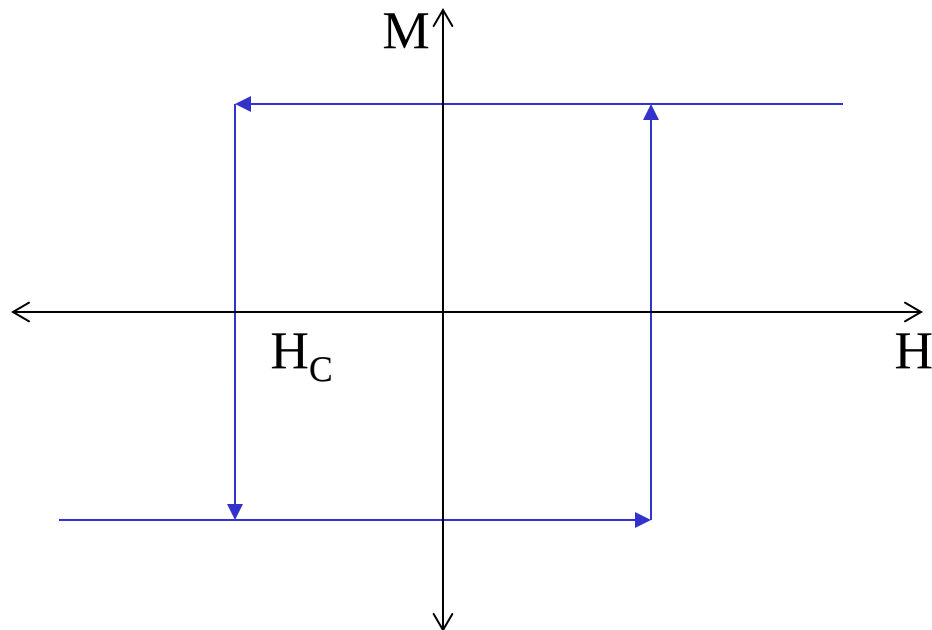
Superparamagnetism →
ultimate limit for storage density

Magnetic Reversal- single domain

Single Magnetic domain (uniaxial)-Stoner Wohlfarth model
(source of anisotropy could be shape)

$$U = K\sin^2(\theta) - M.H$$

$$H_C = 2K/M$$



In practice H_C is smaller than this, e.g. for a curling mode.

Magnetic reversal – multiple domains

Pinning of domain walls leads to a coercivity.

If a material is homogeneous, then wall does not have a position dependent energy, the wall may move freely and the coercivity is close to zero.

Research Area's

- Elemental Rare-Earth Particles and Layers
- Nanostructured Permanent Magnets
- Nanostructured Superconductor/Magnets

Questions you can now answer!

Why does mean field theory work better if atoms have lots of nearest neighbors or if the interaction is long range?

Why do heavy rare-earths have a bigger μ than light rare-earths?

Why do transition metals have a smaller μ than rare-earths?

Why is the magnetic ordering temperature of transition metals larger than for rare-earths?

Why is the magnetic anisotropy of transition metals smaller than rare-earths?

Why do magnets break up into magnetic domains?

What determines the width of a magnetic domain wall?