Effects of primordial kinetic and magnetic helicity

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2005, KSU
Primordial Helicity Generation

- **Cosmological Sources**
  Cornwall, 1997; Giovannini, 2000; Field and Carroll, 2000; Vachaspati 2001; Sigl 2002.

- **MHD Processes in Astrophysical Plasma**
  Vishniac and Cho, 2001; Brandenburg and Blackman, 2002; Subramanian, 2003; Vishniac, Lazarian and Cho, 2003; Subramanian and Brandenburg, 2004; Banerjee and Jedamzik, 2004

- **Turbulence**
  Christensson, Hindmarsh, and Brandenburg, 2002; Verma and Ayyer, 2003; Boldyrev, Cattaneo and Rosner 2005;
Specific cosmological effects

- **Circularly polarized relic gravitational waves**
  * High degree of polarization (for maximal helicity up to 100%), but below current GW missions (LISA) sensibility
  
  Kahniashvili, Gogoberidze and Ratra, 2005

- **Parity odd cross correlations of the CMB fluctuations**
  * Temperature – B-polarization
  * E- and B-polarization

  Pogosian, Vachaspati, and Winitzki, 2002;
  Caprini, Durrer, and Kahniashvili, 2004;
  Kahniashvili and Ratra, 2005
Relic Gravitational waves

Generation mechanisms – cosmological GWs

Quantum fluctuations;
Defects; Bubbles motions, and collisions during PT;
Magnetic Fields;
Primordial Plasma Turbulence

In our case – helical turbulent motions
Primordial turbulence

At some moment in the early Universe $t_\star$:

$\rho_{\text{vac}} \rightarrow \rho_{\text{turb}}$

with efficiency $\kappa$, over time $\tau_{\text{stir}}$ on scale $L_S(\propto H^{-1}(t_\star))$


The turbulent energy cascades from larger to smaller scales;
Turbulence is produced in a time much smaller than Hubble time:

**GWs Equation**

\[ \ddot{h}_{ij}(k, t) + k^2 h_{ij}(k, t) = 16\pi G \ \Pi_{ij}(k, t), \]

we define: \( h_{ij} = \delta g_{ij}, \ h_{ii} = 0 \) and \( h_{ij}k^{ij} = 0. \)

we use natural units \( \hbar = 1 = c, \) and physical/proper \( k \) and physical time \( t. \)
★ Plasma motion consists on kinetic energy simultaneously presented in all scales within the inertial interval $k_S < k < k_D$ ($k_S = 2\pi/L_S$, $k_D = 2\pi/L_D$), the range of the scales without source and sink of turbulent fluctuations and lasting for a total time $\tau \equiv \max(\tau_{\text{stir}}, \tau_S)$, where $\tau_S$ the turnover time. The total free energy density is being injected continually over the time $\tau$ rather than an impulse.

★ Unequal time correlations

$$\langle u^*_i(k, t) u_j(k', t') \rangle = (2\pi)^3 \delta^{(3)}(k - k') [P_{ij} F_S(k, t - t') + i \epsilon_{ijl} \tilde{k}_l F_H(k, t - t')]$$
The decay of the hydrodynamic turbulence in the inertial interval $F(k,t) = P(k)D(t)$, $D(t)$ is some monotonically decreasing function [Hinze 1975]. Extending this assumption for helical turbulence we model $F_S(k,t) = P_S(k)D_1(t)$ and $F_H(k,t) = P_H(k)D_2(t)$ for $k_S < k < k_D$. $D_1(t), D_2(t)$ are monotonically decreasing functions.
In the inertial interval $P_S(k) \sim k^{n_S}$ and $P_H(k) \sim k^{n_H}$. Non-helical hydrodynamic turbulence, [Kolmogorov 1941] $n_S = -11/3$, $E(k) = C_k \bar{\varepsilon}^{2/3} k^{-5/3}$, where $C_k \approx 1$ $\bar{\varepsilon}$ -energy dissipation rate per unit enthalpy. Magnetized medium Iroshnikov-Krichnan $n_S = -7/2$ is possible. Helical hydrodynamic turbulence, two possible spectra associated with: the coexisting cascade of energy and helicity, HK $n_S = -11/3$, $n_H = -14/3$ [Lesieur 1997], and the transfer of helicity HT $n_S = n_H = -13/3$ [Moiseev and Chkhetiani 1996]. If $|P_H(k)| \ll P_S(k)$ only HK is observed in the inertial range [Borue and Orszag 1997].
GW are much more effectively generated by large-scale turbulent fluctuations. [Kosowsky, Mack, and Kahniashvili 2002, Dolgov, Grasso, and Nicolis 2002], so we approximate that simple power-law spectrum seems reasonable model for helical cosmological turbulence,

\[ P_S(k) = S_0 k^{n_S} \quad \text{and} \quad P_H(k) = A_0 k_S^{n_S-n_H} k^{n_H} \]

\* HK: \( S_0 = \pi^2 C_k \bar{\varepsilon}^{2/3} \), \( A_0 = \pi^2 C_k \bar{\delta}^{2/3} / k_S \) [Ditlevsen and Giuliani 2001], \( \rightarrow A_0/S_0 = \bar{\delta} / \bar{\varepsilon} k_S \);

\* HT: \( S_0 = \pi^2 C_s \bar{\delta}^{2/3} \), \( A_0 = \pi^2 C_a \bar{\delta}^{2/3} \) [Moiseev and Chkhetiani 1996], \( \bar{\delta} \) is mean helicity dissipation rate per unit enthalpy, \( C_s, C_a \approx 1 \) for max. helicity \( C_s \sim C_a \).
Since the turbulent motions model is being specified

we are READY

to compute gravitational wave power spectrum
Step 1 - convolution

From $\langle u_i^*(k, t) u_j(k', t') \rangle \rightarrow$

we can get $\langle \Pi_{ij}^*(k, t) \Pi_{lm}(k', t + y) \rangle$

the turbulent source power spectrum

symmetric part $f(k, y) \leftarrow$

$$\int d^3p \left[ F_S(p, y) F_S(|k - p|, y) \phi_S(\hat{k}, \hat{p}) 
- F_H(p, y) F_H(|k - p|, y) \phi_H(\hat{k}, \hat{p}) \right]$$

helical part $g(k, y) \leftarrow$

$$\int d^3p F_S(p, y) F_H(|k - p|, y) \phi(\hat{k}, \hat{p})$$
Step 2 - time integration

The time integrals are: over $f(k,y)$ and $g(k,y)$, against the functions $\cos(ky) D_{1,2}(y)$. Both $D_1(y)$ and $D_2(y)$ are positive functions decreasing with respect of $y$, → the integrals also are oscillatory.

The $\cos(ky)$ terms will always oscillate on time scale shorter than the characteristic time for $D(y)$

$$
\int_{t_{in}}^{t_{end}} dy \cos(ky) F_a(p,y) F_b(|k - p|, y) \approx \sqrt{2} P_a(p) P_b(|k - p|)/(2k)
$$
Step 3 - turbulence – GW

Actually $\int d^3p_1 \int d^3p_2$ integration

For $n_S < -3$ lower cut-off scale region $k_0 \sim k_S$ is dominant, $\rightarrow$ influence of $k_D$ is negligibly small, resulting in insensitivity of GW spectrum and $P$ on $k_D$ for both HK and HT turbulence.
Results

For turbulence with maximal helicity, $A_0 = S_0$, $n_H = n_S < -3$, $\mathcal{P}(k) \approx 1$ for $k_S < k < k_D$

For weakly helical turbulence $A_0 \ll S_0$, $n_H \approx n_S < -3$, $\mathcal{P}(k) \rightarrow CA_0/S_0$, $1 < C(n_S, n_H) < 2$ is constant determined by $n_S$ and $n_H$: for $n_S = n_H = -13/3$, $C \approx 1.5$; For Iroshnikov-Krithnan $n_S = n_H = -7/2$, $C \approx 1.39$

For arbitrary spectral indices $n_S$, $n_H < -3$, $H(k)$ and $\mathcal{H}(k)$ behave as $\propto k^{n_S-3} k_S^{n_S+3}$, $\rightarrow$ in this limit $\mathcal{P}(k)$ is approximately constant with respect to $k$. 
Expected GW energy density parameter today

\[ \Omega_{GW}(f)h^2 \simeq 1.05 \times 10^{-11} \left( \frac{L_S^2}{\tau H_*^{-1}} \cdot \frac{n_S + 5}{|n_S + 3|} \right)^2 \left( \frac{L_D}{L_S} \right)^{3(n_S+5)} \left( \frac{3 \kappa \rho_{vac} L_S}{4 \nu \rho_{rad}} \right)^3 \left( \frac{f}{f_S} \right)^{\frac{2(2n_S+5)}{n_S+5}} g_{100}^{-\frac{1}{3}} \]

\[ f_S = 1.9 \times 10^{-6} \text{Hz} \sqrt{\frac{n_S + 5}{|n_S + 3|}} \left( \frac{3 \kappa \rho_{vac}}{4 \rho_{rad}} \right)^{\frac{1}{2}} \left( \frac{\tau \nu}{H_*^{-2}} \right)^{-\frac{1}{2}} \left( \frac{L_D}{L_S} \right)^{\frac{n_S+5}{2}} \left( \frac{T_*}{100 \text{Gev}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}} \]
Relic gravitational waves background

- **Theoretically possible test:**
  circular polarization degree of relic gravitational waves background! \( P(k) \) strongly depends on \( P_{H0}/P_{B0} \)

- **Requirements:**
  at least two antennas;
  high angular resolution and sensibility
How could we test primordial magnetic helicity?

• **Direct test**
  
  *(Faraday Rotation)*
  
  🔄 **NO**
  Ensslin and Vogt, 2003; Campanelli et al., 2004; Kosowsky, Kahniashvili, Lavrelashvili, and Ratra, 2005

• **Un-direct test**
  
  *(through induced specific effects)*

  🔄 **Difficult, BUT possible**
  Kahniashvili and Ratra, 2005
Magnetic field statistical properties

Stochastic Gaussian helical magnetic field

\[ \langle B_i^*(k)B_j(k') \rangle = (2\pi)^3 \delta^{(3)}(k - k') [P_{ij}(\hat{k}) P_B(k) + i\epsilon_{ijl} \hat{k}_l P_H(k)]. \]

for \( k < k_D \) – damping scale

\[ P_B(k) \equiv P_{B0} k^{n_B} = \frac{2\pi^2 \lambda^3 B^2}{\Gamma(n_B/2 + 3/2)} (\lambda k)^{n_B}, \]

\( B_\lambda^2 \) - energy density

\[ H_\lambda^2 \] – mean helicity

\[ P_H(k) \equiv P_{H0} k^{n_H} = \frac{2\pi^2 \lambda^3 H^2}{\Gamma(n_H/2 + 2)} (\lambda k)^{n_H} \]
Helical magnetic source - assumptions

* Infinite conductivity of primordial plasma

\[ B(t, x) = \frac{B(x)}{a^2} \]

* Causality requirement

\[ P_B(k) \geq |P_H(k)| \]

* Non-divergence of energy density and average helicity on super-horizon scales

\[ n_B > -3 \quad n_H > -4 \]
Metric Perturbations from Magnetic field

\[ G_{ik} = 8\pi G T_{ik} \]

- Scalar mode (density perturbations)–
  no contribution from magnetic helicity
  into the scalar part of the stress-energy tensor

- Vector (vorticity perturbations, Alfvén waves)
  Non-zero contribution! CMB anisotropies

- Tensor (gravitational waves)
CMB temperature and polarization anisotropies

Hu and White 1997

\[ C^{XX'}_l = \frac{2}{\pi} \int dk k^2 \sum_m \frac{X_l(m)(\eta_0, k)}{2l + 1} \frac{X'_l(m)(\eta_0, k)}{2l + 1} \]
CMB anisotropies power spectra: helicity signatures

- Parity-even power spectra: additional effects
  \[ C_l^{XX'} = C_{(S)l}^{XX'} - C_{(A)l}^{XX'} \]

- Parity-odd power spectra:
  Parity symmetry breaking! \[ C_l^{\Theta B} \] and \[ C_l^{EB} \]
  non-vanishing correlations
Maximal helicity effects

- Significant reduction for parity-even power spectra (comparing with the non-helical case);

- Comparable (by amplitude) cross correlations between temperature-$E$-polarization and temperature-$B$-polarization;

- Comparable (for large angular scales) cross correlations between temperature-$E$-polarization and $E-B$-polarization;
Vector – Tensor modes comparison

- **Vector mode**
  - Surviving up to small angular scales.
  - Vanishing E-B polarization cross correlations (with respect of temperature-B-polarization)

- **Tensor mode**
  - Gravitational wave source damping after equality!
  - Contribution in CMB for large angular scales (l <100)
  - The same order of magnitude for temperature – B-polarization and E-B polarization cross correlations.
How to constraint primordial magnetic helicity

WARNING

Even for primordial magnetic field with maximal helicity such effects may be detectable if the current magnetic field amplitude is at least $10^{-9} - 10^{-10}$ Gauss on Mpc scales.
Step 1: Symmetric part reconstruction

- Since Faraday rotation measurement is independent on magnetic helicity, the amplitude and configuration of a primordial magnetic field could be obtained through CMB polarization plane RM;

- B-polarization measurement ! peak position ! identification of the source for B-polarization !

  \[ <|RM|^2>^{1/2} \]

Campanelli et al, 2004; Lewis 2004; Kosowsky, et al. 2005
Average rotation angle and B-polarization signal by Faraday rotation

Kosowsky et al., 2005

FIG. 1: The angular power spectrum of the Faraday rotation angle, $C_l^\theta$, induced by a stochastic magnetic field. The curves in order of decreasing amplitude on the right side of the plot correspond to magnetic field power spectral indices $n = 2, 1, 0, -1,$ and $-2$. The magnetic fields have been normalized to a nanogauss at the smoothing scale $\lambda = 1$ Mpc.

FIG. 2: The C-polarization power spectrum of the microwave background induced by the Faraday rotation field in Fig. 1, with the magnetic field normalization scale $\lambda = 1$ Mpc.
Step 2: Spectral indices $n_B$ and $n_H$

- For small angular scales cross-check of $C_l^E B \ll C_l^{E B}$.

- Full maps of the CMB temperature and polarization anisotropies (parity-even power spectra).

\[
\kappa_{\chi \chi'} = 1 - \frac{2(2n_B + 3)}{3(2n_H + 3)} \left( \frac{P_{H0} k_D^{n_H-n_B}}{P_{B0}} \right)^2 \mathcal{R}_{\chi \chi}(n_B, n_H, l)
\]
Step 3: Average helicity magnitude

- Measurement of temperature-B-polarization cross-correlations on small angular scales with prior of magnetic field amplitude and spectral indexes

average helicity constraint
Conclusion

• Primordial helicity affects the generation of the CMB fluctuations
• Primordial helicity generates circularly polarized gravitational waves
• Such effects are possibly detectable, but a high precision of measurement is required