Neutrino properties

Glenn Horton-Smith
KSU 4:30 Wednesday Math Seminar
Part I: April 13, 2011
Part II: April 20, 2011

Outline of Known Properties

- Three light neutrino “flavors”: ν_e, ν_μ, ν_τ
  - All three are at least 5 powers of 10 less massive than the electron. (Cosmologists say at least 6.)
- Only see interactions of left-handed neutrinos (ν_L), right-handed anti-neutrinos ($\bar{\nu}_R$)
- No evidence for ν_R or $\bar{\nu}_L$.
  - If they exist, then they are “sterile”, or very heavy, or both.
- Mixing of the three neutrino flavors in vacuum
- At least two mixing modes: ν_1-ν_2 seen in ν_e , ν_2-ν_3 in ν_μ

Outline of Unknown Properties

- Is there mixing of 1-3, and if so, how much? ***
- Are there additional (sterile) light neutrinos? ***
- What is the mass hierarchy of the neutrinos? **
- Is there a Majorana mass term for neutrinos? *
- Are there heavy (possibly RH) neutrinos?

(The number of *s indicates how much I know about the experiments that are supposed to address these issues.)

Key Place for Review of Particle Properties

- http://pdg.lbl.gov -- Particle Data Group
- Publishes the *Review of Particle Physics*.
  - Citation for latest published version: K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010) .
- Chapter 13: Neutrino Mass, Mixing, and Oscillations
  - Completely rewritten by new authors in most recent edition!
- Also has tables of experimental results, and separate articles for “Neutrinoless Double-Beta Decay” and “The Number of Light Neutrino Types”.

Outline of the neutrino chapter from the *Review* (for comparison)

. NEUTRINO MASS, MIXING, AND OSCILLATIONS

. 1. Introduction: Massive neutrinos and neutrino mixing

. 2. Neutrino oscillations in vacuum

. 3. Matter effects in neutrino oscillations (in Earth and Sun)

. 4. Measurements of solar Δm^2 and θ

. 5. Measurements of atmospheric $|\Delta m^2|$ and θ

. 6. Measurements of θ_{13}

. 7. The three neutrino mixing, including the “see-saw mechanism”, the baryon asymmetry of the Universe, and the nature of massive neutrinos

. 8. Outlook

What I will try to summarize

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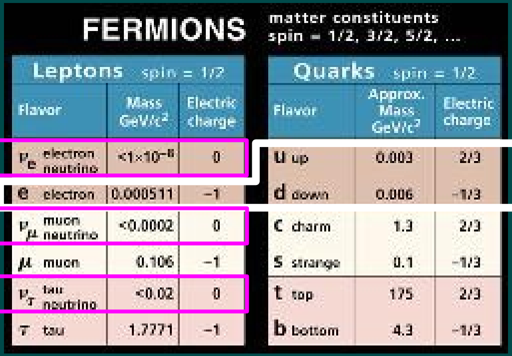
Neutrinos as elementary particles



FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

BOSONS force carriers spin = 0, 1, 2, ...

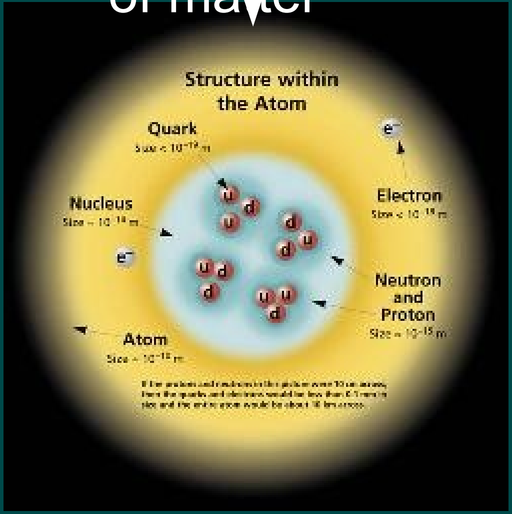
Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

stable
constituents
of matter

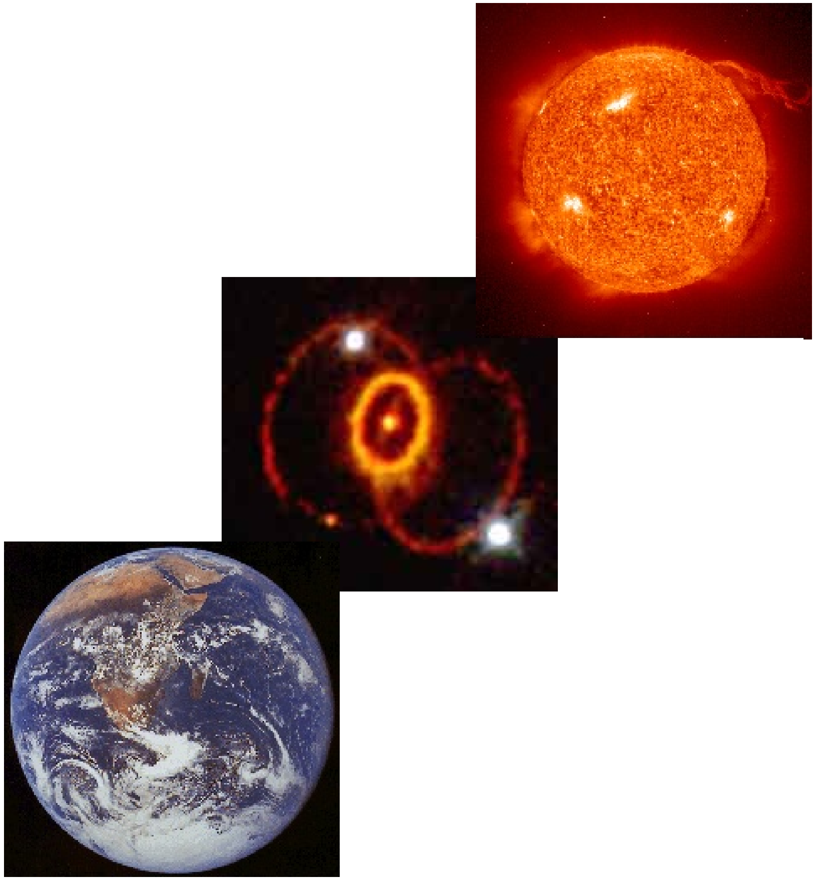


Neutrinos are very different from other elementary particles. ***Why?***

Knowing more about them should help.

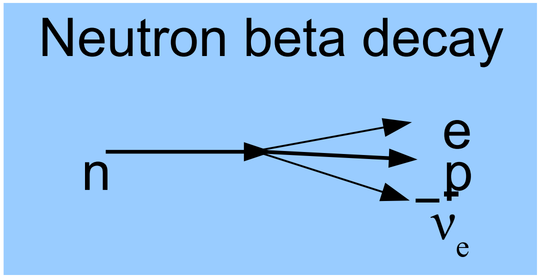
Neutrinos

- Neutrinos have been seen from the sun, from nuclear power stations, from cosmic rays, from accelerators, and even from a supernova. Because they are almost “unstoppable”, we can use them to...
  - ... peer inside the Sun.
  - ... detect supernovae and other exciting things.
  - ... radioassay a huge sample of the Earth’s crust.



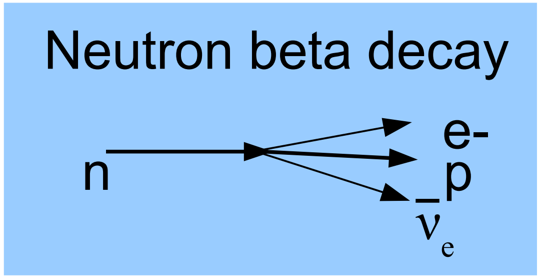
Neutrino interactions

- The neutrino is the most difficult to detect particle that has ever been directly observed.
- Neutrinos are emitted in certain nuclear and particle decays.
- Neutrinos hardly interact with matter at all: a typical anti-neutrino from radioactive decay can pass through ~10^{14} km (~10 light years) of lead.

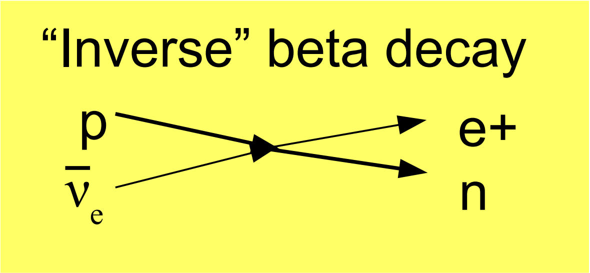

Neutron beta decay

How can we detect neutrinos, even in principle?

- Although they hardly interact at all, they *do* interact.
- For any interaction or decay that makes a neutrino, there is a corresponding one that absorbs one.
- There are also neutrino scattering interactions.



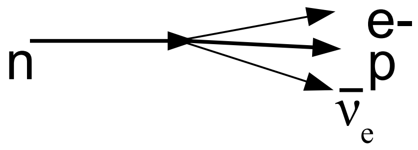
Neutron beta decay

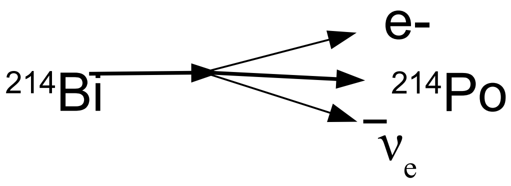


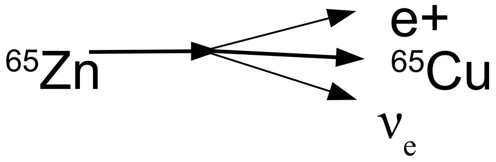
“Inverse” beta decay

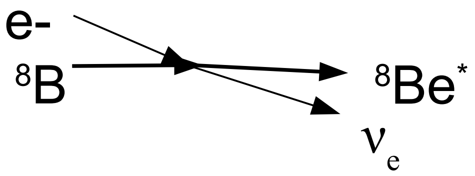
More interactions of the neutrino

Beta decays & electron capture

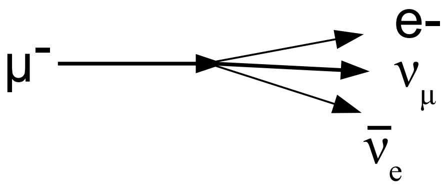




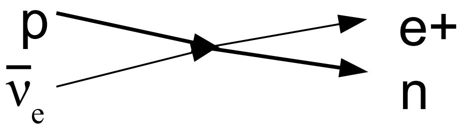




“Beta decay” of a muon



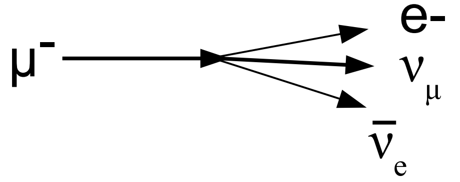
“Inverse” beta decay



Existence of “flavors”

- Notice all of the interactions conserve the total number of electron-type leptons and the number of muon-type leptons.

“Beta decay” of a muon



- “Lepton numbers” individually conserved:
  - $L_e = N_{e-} + N_{\nu_e} - N_{e+} + N_{\bar{\nu}_e}$
  - $L_\mu = N_{\mu-} + N_{\nu_\mu} - N_{\mu+} + N_{\bar{\nu}_\mu}$
  - Also one for the “tau” lepton flavor.
- If we didn't have separate muon and electron neutrinos, then the muon could decay to an electron + photon.
- Limits on non-conservation in interactions ~10^{-12} level. (See “Tests of Conservation Laws” in *Review*.)

The masses of the known neutrinos

- Direct limits on electron (anti)neutrino masses come from tritium beta decay.
  PDG evaluation: <2 eV, 95% CL.
- Limits on muon and tau neutrino masses come from particle decays: <0.19 MeV and <18 MeV, resp.
- Limits on sum of all neutrino masses come from cosmology, range < 0.2~1 eV depending on model assumptions.

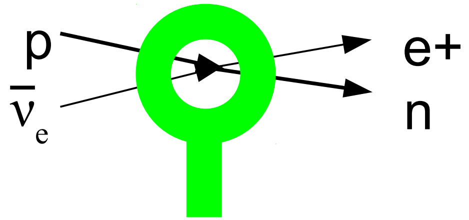
Helicity of the neutrinos

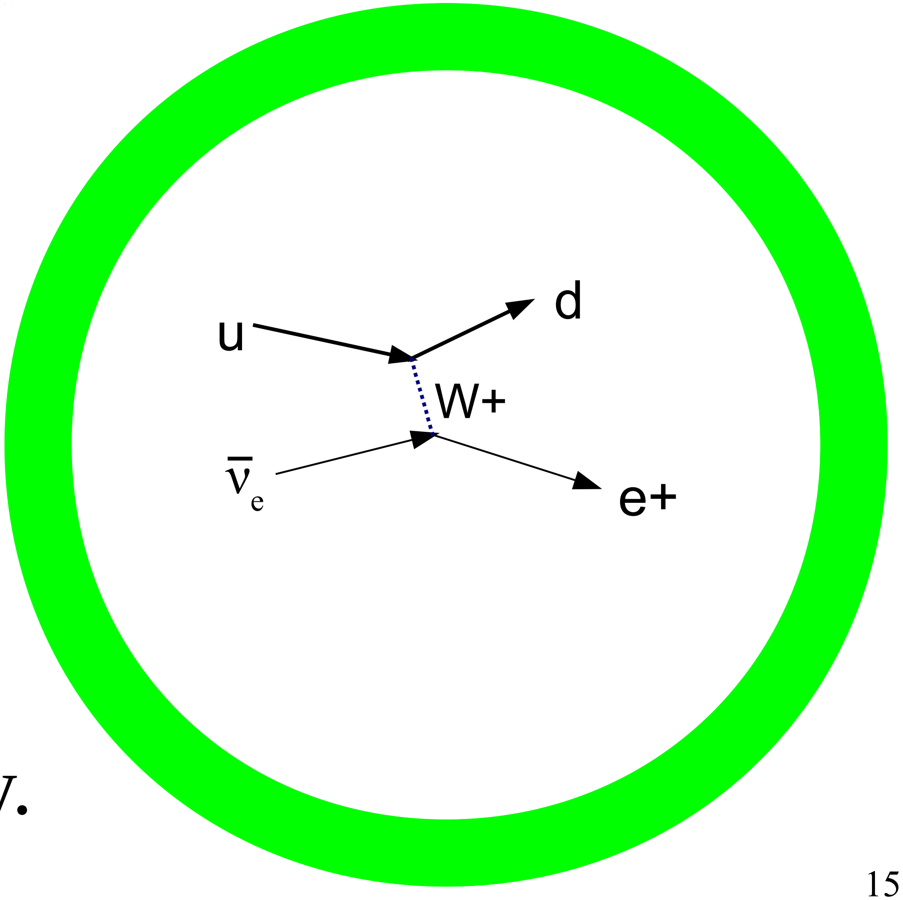
- A fermion (1/2 spin) can have +1/2 or -1/2 spin along the direction of its motion: right-handed (RH) or left-handed (LH) helicity.
- Every observed interaction supports the model that interactions happen only with LH neutrinos or RH antineutrinos.
- Mathematically, this is implemented in the “standard model” by a factor $(1-\gamma^5)$ that zeroes out the amplitude of any interaction with a RH neutrino or LH antineutrino.

A closer look at neutrino interactions

- The interactions of neutrinos with other particles is mediated by the Z and W bosons.

“Inverse” beta decay

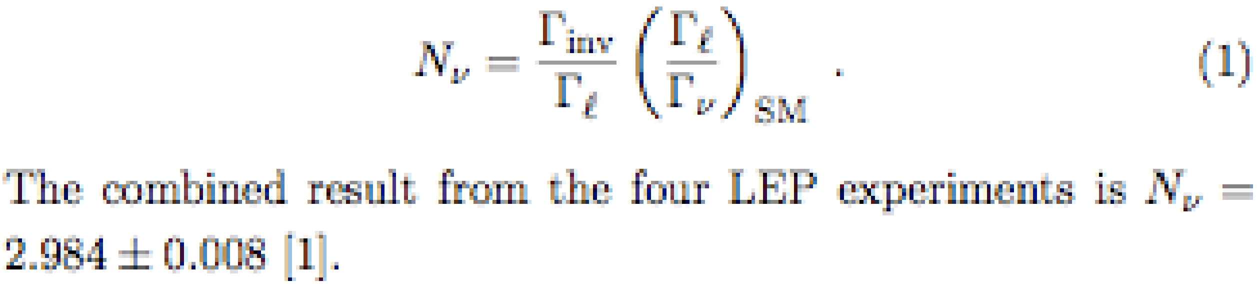




- The high mass of the Z and W result in low interaction cross-sections at low energy.

The number of light flavors

- The width of Z boson decays depends on the number of neutrinos the Z can decay into.
  - Calculate the “invisible” decay mode partial width from the visible decays and the total width.
  - To reduce model dependence, look at ratio of “invisible” to observed leptonic partial width.



$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\ell}\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{\rm SM} \quad . \qquad (1)$$

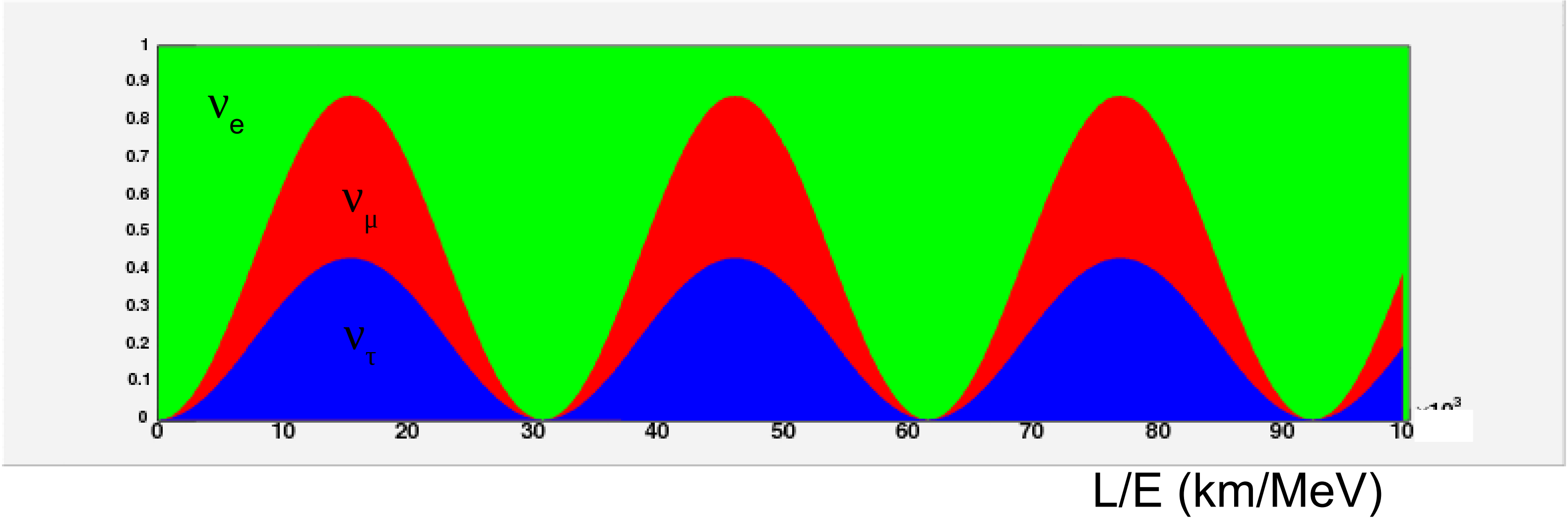
The combined result from the four LEP experiments is $N_\nu = 2.984 \pm 0.008$ [1].

- Result: # neutrinos = 2.984 ± 0.008.
- Only counts neutrinos less massive than the Z.

Neutrino oscillations

In vacuum, a beam of pure electron neutrinos will seem to “disappear” and reappear according to:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m_{12}^2 L}{4 E_\nu}$$



Dense matter can change the effective values of θ and Δm^2 for electron neutrinos. (But not other neutrinos.)

Neutrino oscillations in vacuum (basic QM theory)

- This is attributed to the lepton flavor states being a mixture of the neutrino mass states.

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

The two-neutrino system

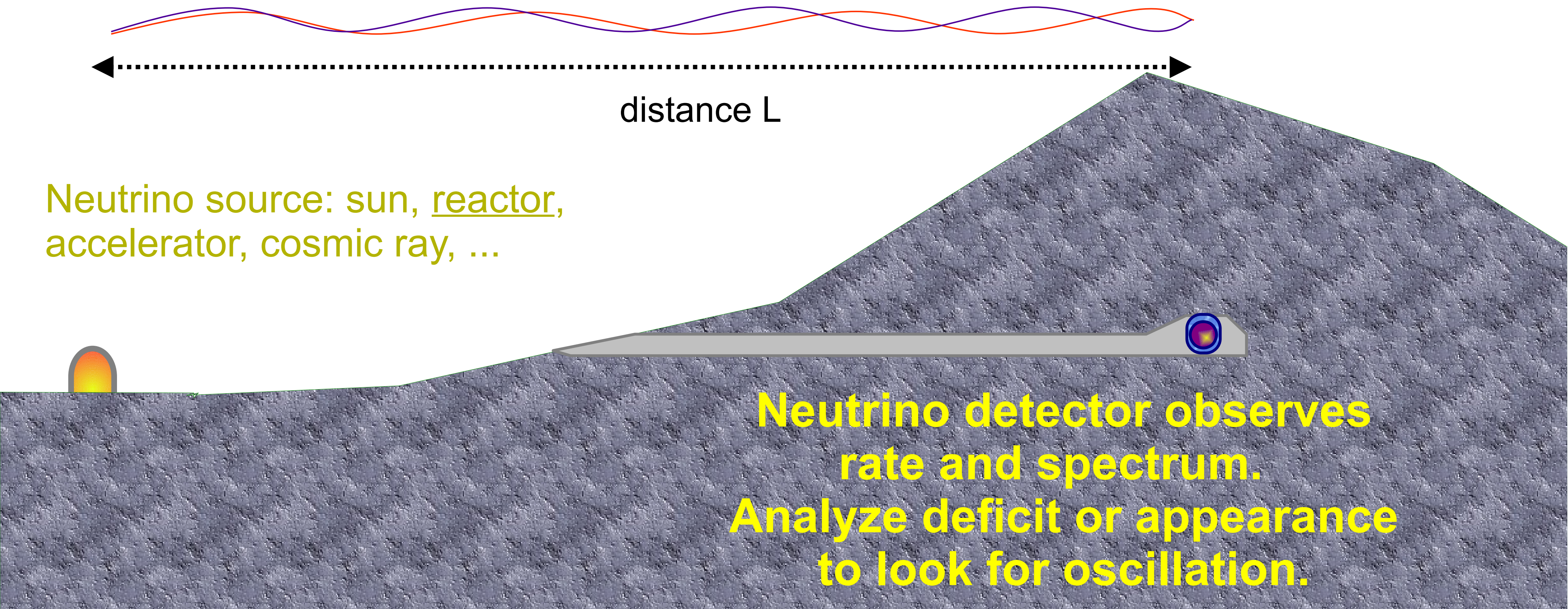
$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} |\nu_1> \\ |\nu_2> \end{bmatrix}$$

$$P_{\nu_e \to \nu_e}(L) = |\langle \nu_e(L) | \nu_e(0) \rangle|^2 = 1 - \sin^2(2\theta)\sin^2\left(\frac{(m_2^2 - m_1^2)L}{4E}\right)$$

- The neutrino state that interacts solely with electrons, ν_e, is a mixture of mass states.
- This leads to beats as the neutrino propagates.

Oscillation between source and detector

$$P_{\bar{\nu}_e \rightarrow not\ \bar{\nu}_e} = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

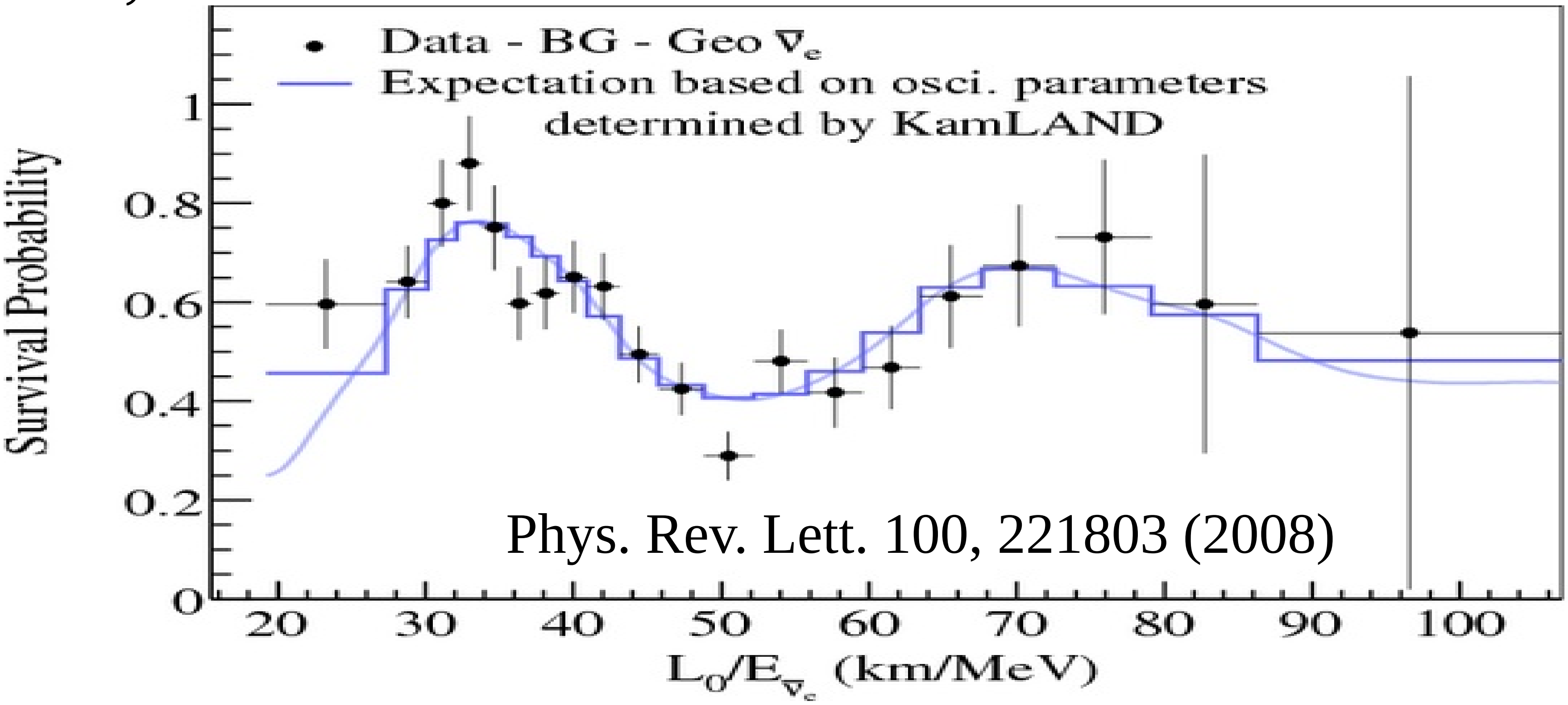


Neutrino source: sun, reactor, accelerator, cosmic ray, ...

Neutrino detector observes rate and spectrum. Analyze deficit or appearance to look for oscillation.

Evidence for two-neutrino oscillation (“solar” type)

- **KamLAND** sees a clear oscillation signature in electron antineutrinos from many reactors at L~180 km, E=1.8~8 MeV.

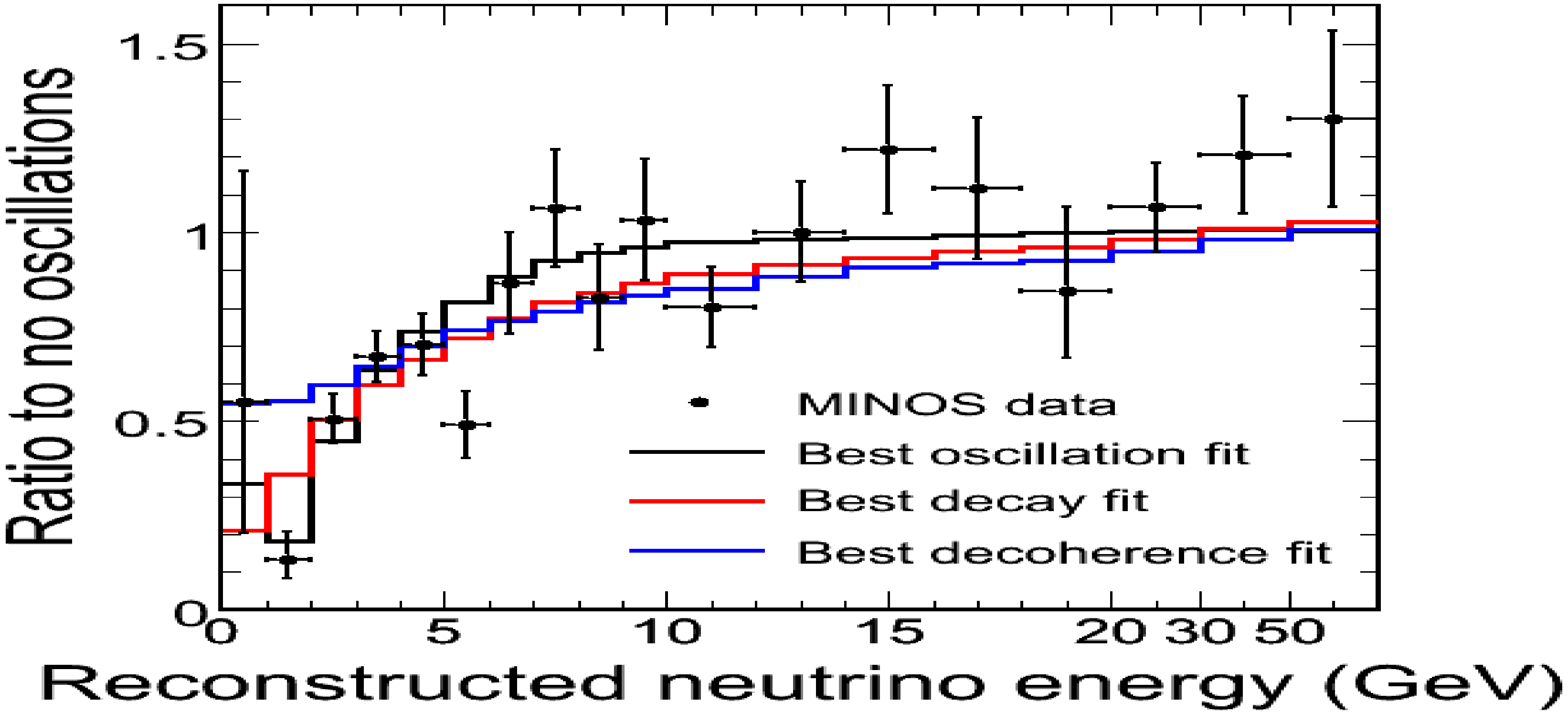


Phys. Rev. Lett. 100, 221803 (2008)

- Also, SNO sees effect on solar ν_e.

Evidence for another “two-neutrino” oscillation mode (“atmospheric” type)

- MINOS and Super-K see clear “disappearance” of muon neutrinos at L=735 km, E=1~50 GeV. Oscillation favored (3.7σ).



MINOS http://www-numi.fnal.gov/PublicInfo/forscientists.html, 2009/09/19.

The “bi-mixing” matrix

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = R_1(\theta_{23})\,R_3(\theta_{12})\begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\cos\theta_{23}\sin\theta_{12} & \cos\theta_{23}\cos\theta_{12} & \sin\theta_{23} \\ \sin\theta_{23}\sin\theta_{12} & -\sin\theta_{23}\cos\theta_{12} & \cos\theta_{23} \end{bmatrix}\begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

- Two “rotations”: one about the 3 axis, and another about the resulting “1” (e) axis.
- θ_{23} = 45°±6°, θ_{12} = 34.5°±1.3°.
  - Compare to quarks: θ_{us} = 12.9°±0.2°, θ_{cb} = 2.4°±0.1°.

Break

Neutrino properties II

Glenn Horton-Smith
KSU 4:30 Wednesday Math Seminar
April 20, 2011

Mixing matrix with two mixing angles

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = R_1(\theta_{23})\, R_3(\theta_{12}) \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\cos\theta_{23}\sin\theta_{12} & \cos\theta_{23}\cos\theta_{12} & \sin\theta_{23} \\ \sin\theta_{23}\sin\theta_{12} & -\sin\theta_{23}\cos\theta_{12} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

- Two “rotations”: one about the 3 axis, and another about the resulting “1” (e) axis.
- $\theta_{23} = 45° \pm 6°$, $\theta_{12} = 34.5° \pm 1.3°$.
  - Compare to quarks: $\theta_{us} = 12.9° \pm 0.2°$, $\theta_{cb} = 2.4° \pm 0.1°$.

This is pretty close to the so-called "tribimaximal" matrix

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

- "Trimaximal" in equality of m_2 components of all three flavors.
- "Bimaximal" in full mixing of 2 and 3 in mu/tau oscillations.

The general mixing matrix, without complex phases

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = R_1(\theta_{23}) R_2(\theta_{13}) R_3(\theta_{12}) \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

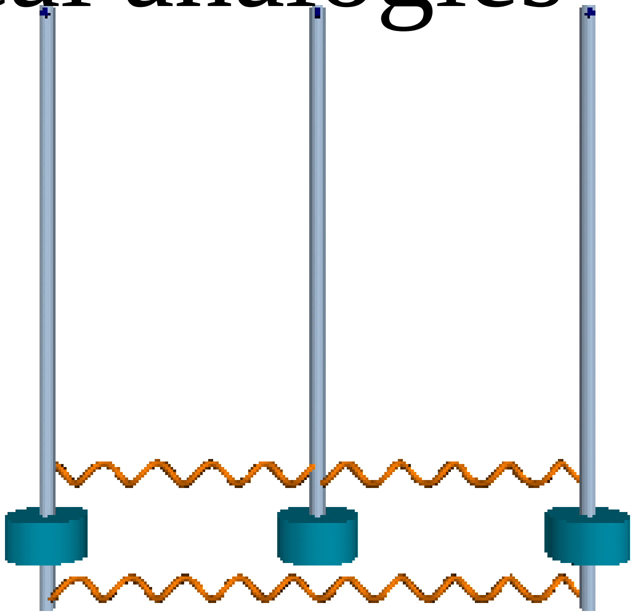
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & \sin\theta_{13} \\ -c_{23}s_{12}-c_{12}s_{13}s_{23} & c_{12}c_{23}-s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}c_{23}s_{13} & -c_{12}s_{23}-c_{23}s_{12}s_{13} & c_{13}c_{23} \end{bmatrix} \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

Here $c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$

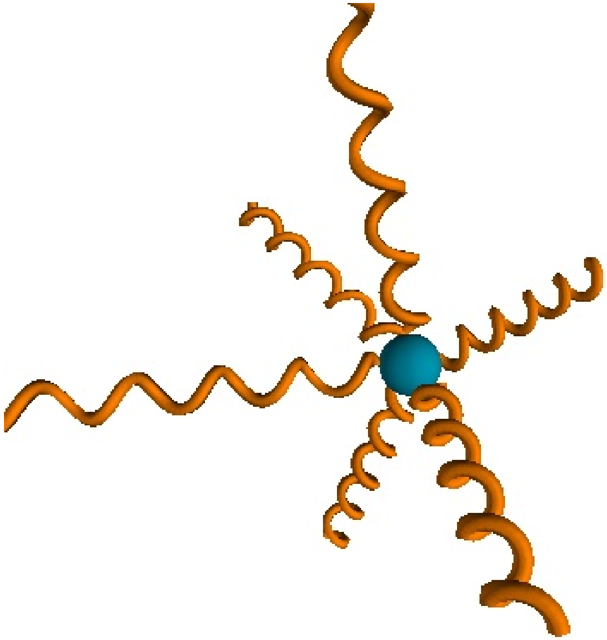
- $\sin\theta_{13}$ represents amount of ν_3 in ν_e.

Three-component mechanical analogies

- Three coupled pendulums
  - Coupling through springs
  - Only two of three modes move central mass. (θ_{13}=0)



- 3d mass-on-springs
  - “Coupling” through rotation
  - Arbitrary rotation would couple all three. (θ_{13}= anything)



Existing 90% limits on sin θ_{13}

Experiment	$\sin^2 2\theta_{13}$	$\sin \theta_{13}$
CHOOZ	<0.19	<0.22
Palo Verde	<0.36	<0.32
MINOS	<0.42	<0.35
Global fit to all neutrino data	0.06 ± 0.04 or <0.12	0.12 ± 0.04 or <0.18

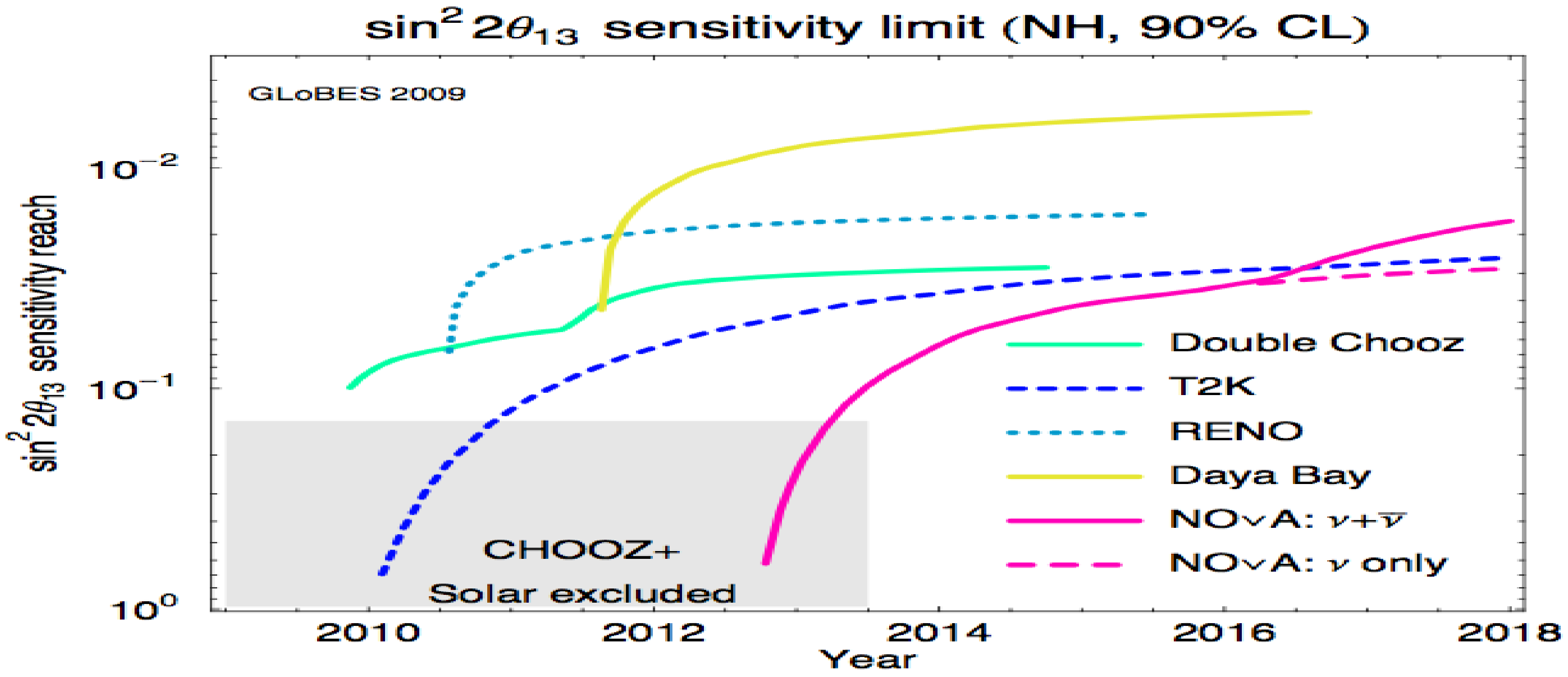
- Experiments measure $\sin^2 2\theta_{13}$.
- Best direct limit is from CHOOZ (1999): a 5 ton detector at 1 km from two 4.3 GWth reactors.

Some new experiments to study θ_{13}

Experiment	Mass and power	$\sin^2 2\theta_{13}$ 90% limit
Double Chooz	10t, 8.6 GWth	0.10 (Phase 1) 0.03 (Phase 2)
Daya Bay	80t, 18 GWth	0.01
RENO	30t, 16 GWth	0.02
T2K, NOνA	muon neutrinos	0.004~0.02

- **Double Chooz**, Daya Bay, and RENO are multi-detector, multi-reactor experiments.
- T2K and NOνA are experiments using muon neutrinos made by accelerators, like MINOS.

Projected sensitivity vs time of new experiments to study θ_{13}



- Plot by Huber, et al, arXiv:0907.1896v1, July, 2009, based on information from each collaboration. Move **Double Chooz** line ~0.5 year to the right, Daya Bay a little more; move others...?

Beyond θ_{13}: complex phases

- Neutrino wavefunctions incorporate complex numbers. A three-neutrino mixing matrix can have up to three complex phases.
- They're commonly called δ_{CP}, α_1, and α_2.

The most general three-neutrino mixing matrix

$$\begin{bmatrix} |\nu_e> \\ |\nu_\mu> \\ |\nu_\tau> \end{bmatrix} = U \begin{bmatrix} |\nu_1> \\ |\nu_2> \\ |\nu_3> \end{bmatrix}$$

$$U = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & \sin\theta_{13}e^{-i\delta} \\ -c_{23}s_{12}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

$$\times\ \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1).$$

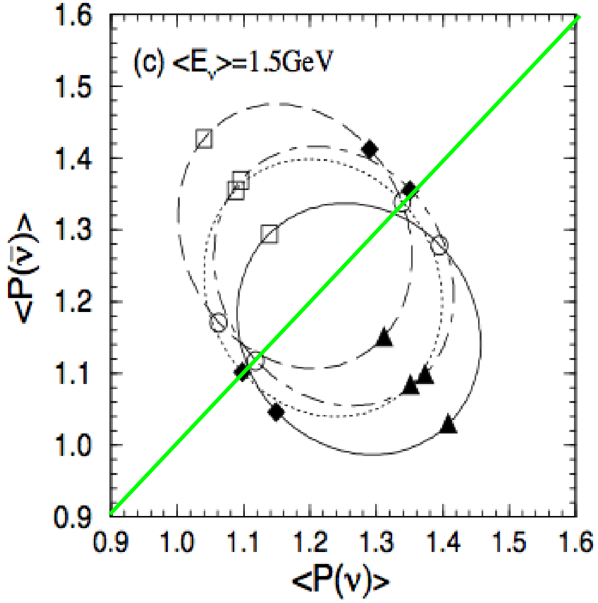
- δ_{CP} breaks “charge-parity” symmetry.
- The α_1 and α_2 are only important if neutrinos can become antineutrinos. (“Majorana mass term”.)

What is “CP”?

- Every particle has a corresponding antiparticle: e+ for e-, $\bar{\nu}$ for ν, etc. If swapping particles for antiparticles doesn't change an interaction, that's C symmetry.
- Particles have spin. If inverting the spin of all particles doesn't change an interaction, that's P symmetry.
  - Remember, active neutrinos are all left-handed, antineutrinos are all right-handed.
- CP is doing both C and P.

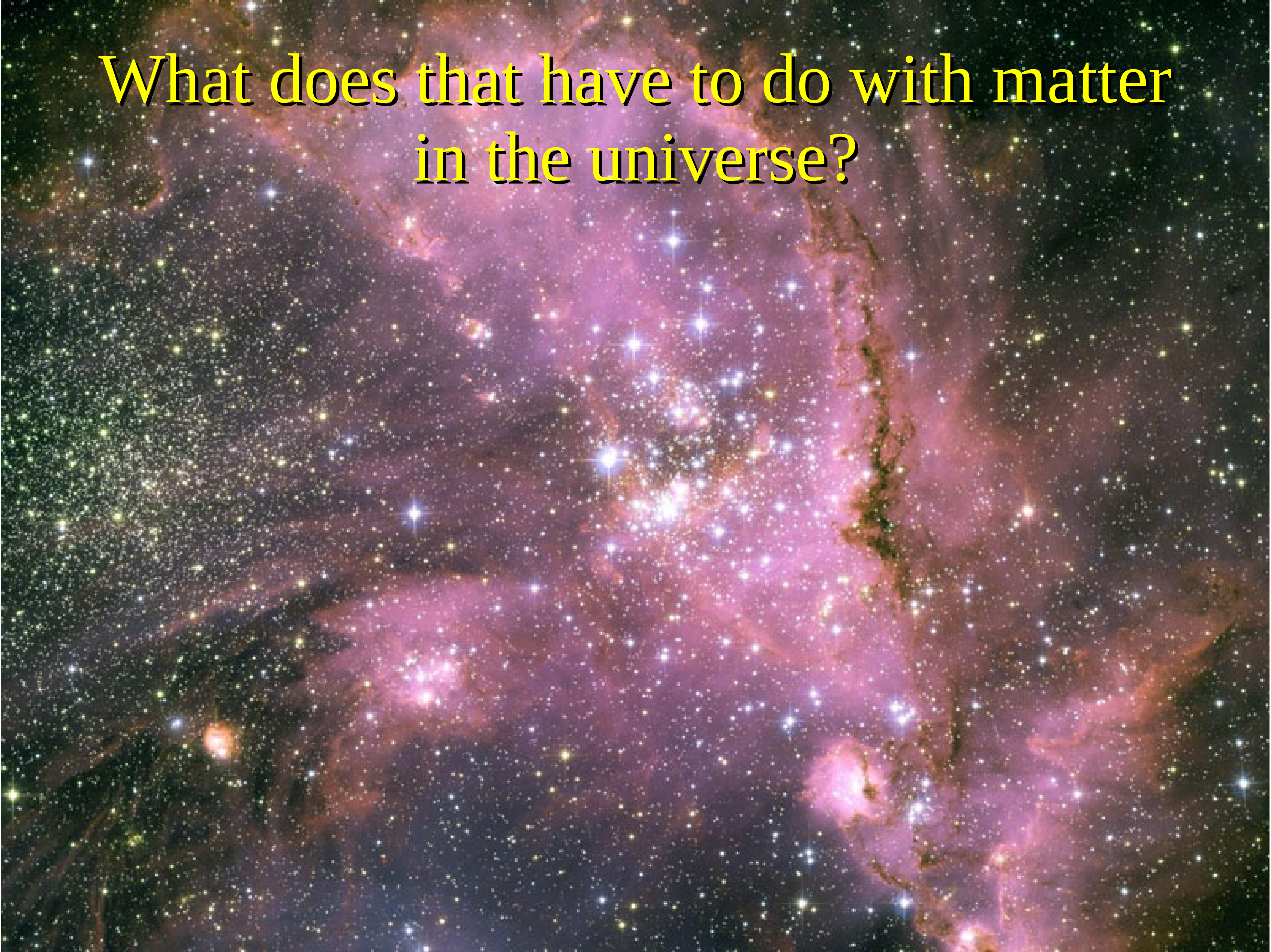
OK, if that's CP, what happens if it's broken?

- δ_{CP} affects antineutrinos differently from how it affects neutrinos.
- Plot at right shows the theoretical probability (in %) in T2K of neutrino oscillation compared to antineutrino oscillation, for various values of δ_{CP} , assuming $\sin^2 2\theta_{13}=0.05$.
- Diagonal green line = no CP breaking.



H. Minakata and H. Nunokawa, JHEP 0110, 001 (2001).

What does that have to do with matter in the universe?



A very quick, simplified review of Big Bang cosmology

- As the universe expands, it cools.
- At high temperatures, $T \gg mc^2$, particle-creating interactions can be in equilibrium with absorption and annihilation.
- This equilibrium disappears as the universe cools. (Neutrinos actually go in and out of equilibrium because it takes them a long time to equilibrate.)
- Similarly, atoms only form once the universe has cooled enough. We see the cosmic microwave background (CMB) from 380,000 years after the Big Bang; before that, the universe was opaque.
- The CMB is so uniform that there must have been a rapid "inflation" earlier.

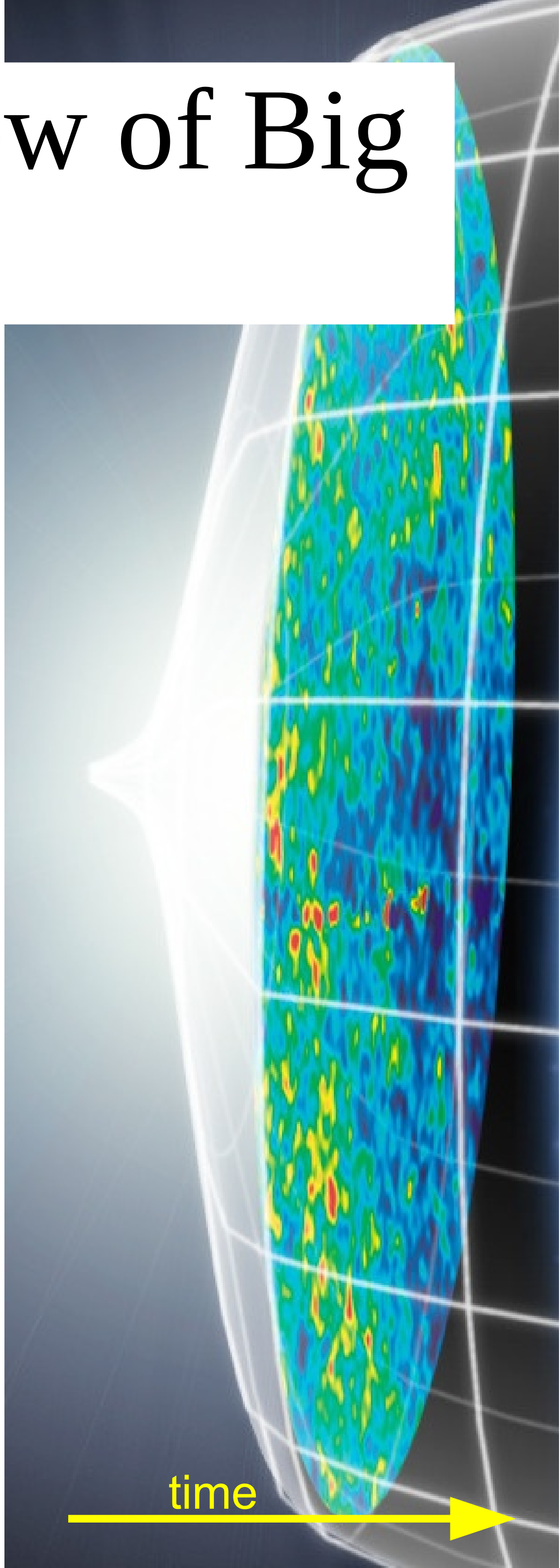
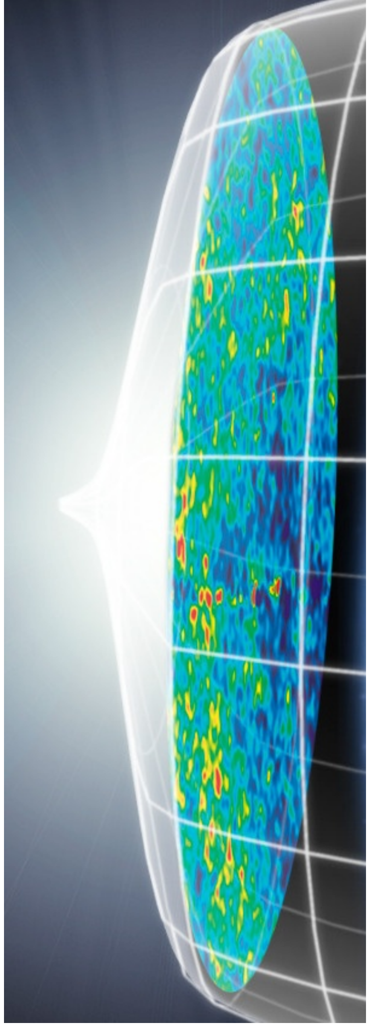


Image credit: WMAP/NASA.

The matter mystery



- The universe is exclusively matter, except for antimatter-matter pairs made in collisions of matter and photons.
- If all interactions during inflation conserved C, CP, and “baryon number”[†], this would not be the case! In fact, any initial matter-antimatter asymmetry would be almost entirely erased.
- We need some particle, out of equilibrium during inflation, with a *large* violation of CP.

† Baryon-number violation is “expected to be contained in grand unified theories as well as in the non-perturbative sector of the Standard Model.” [Review of Particle Physics]

Experiments to measure δ_{CP}

- T2K, “Tokai to Kamioka” (Japan):
  295 km, 0.75 MW beam power.
- NOνA, Fermilab to Minnesota (US):
  810 km, 0.7 MW beam power.
- Good news: Great sensitivity to CP, ordering of neutrino masses, and θ_{13}.
- Bad news 1: sensitive to all three at the same time.
- Bad news 2: if $\theta_{13}=0$, no sensitivity to CP.

Projected "discovery potential" vs time of new experiments

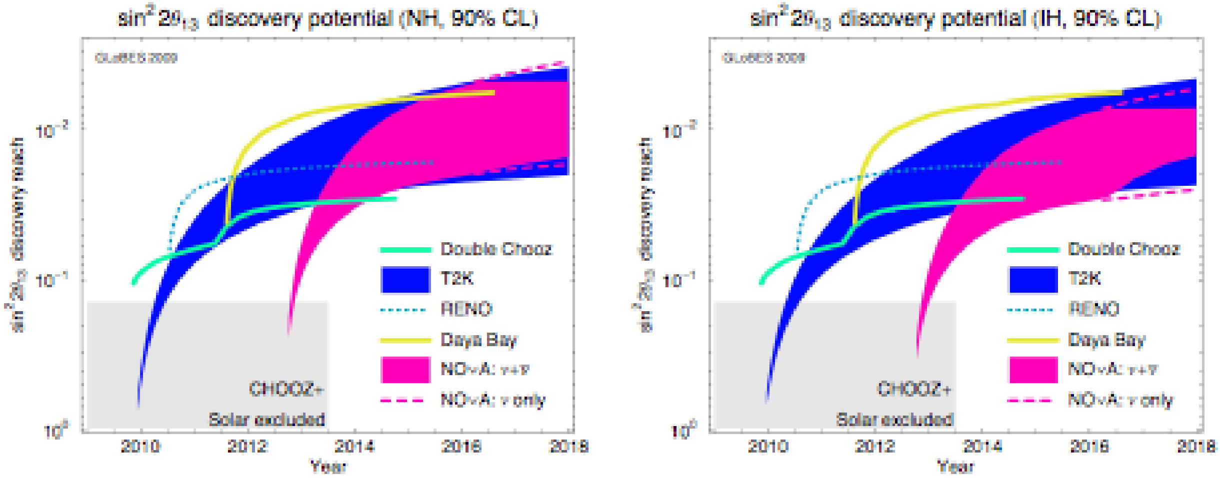
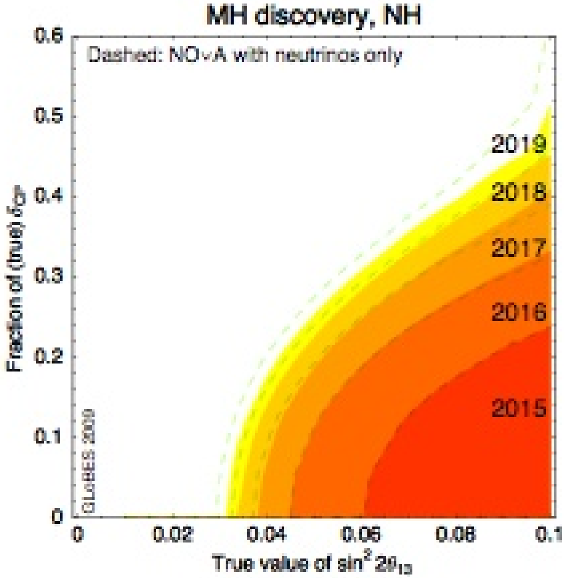


Figure 7: Evolution of the θ_{13} discovery potential as a function of time (90% CL), *i.e.*, the smallest value of θ_{13} which can be distinguished from zero at 90% CL. We assume the normal and inverted simulated hierarchies in the left and right panels, respectively. The bands reflect the (unknown) true value of δ_{CP}.

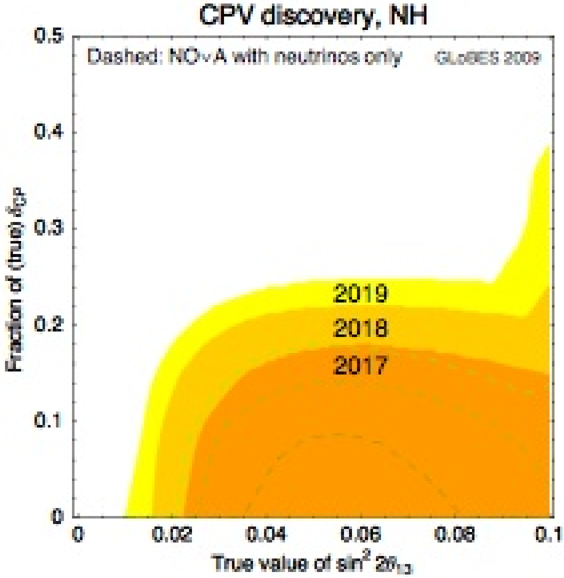
- Again from Huber, et al, arXiv:0907.1896v1, July, 2009.
- Depending on δ_{CP}, T2K and NOvA *might* see a $\theta_{13} \neq 0$ effect, yet not know what θ_{13} is, unless **Double Chooz** or Daya Bay tells them!

Projected “discovery potential” vs time of new experiments

Mass hierarchy “discovery”



CP violation “discovery”



- Again from Huber, et al, arXiv:0907.1896v1, July, 2009. 90% CL. (!) Normal hierarchy plots shown, see original for inverted hierarchy.
- We are very interested to know if $\sin^2 2\theta_{13} < 0.03$.

OK, so let me get this straight...

- If θ_{13} is non-zero, then δ_{CP} has meaning.
- And if so, then neutrinos violate CP.
- And if that effect is big enough, then that would prove that neutrino CP violation is the cause of the matter-antimatter asymmetry of the Universe, right?

Er... not quite that last bit.

The matter-antimatter asymmetry model depends on CP violation in *heavy* neutrinos

- Our familiar, almost-massless neutrinos turn out not to be able to do the job in any model that's been put forth.
- There are “attractive theoretical considerations”[†] that lead one to hypothesize heavy partners to the light neutrinos. (“See-saw model”, etc.)
- CP violation in these heavy neutrinos could do the job.[‡]

† *I.e.*, there is more reason to hypothesize heavy neutrinos than just lack of proof that they don't exist. ‡ No proof that it's not so, yet.

The see-saw mechanism

1. Add an explicit mass term only for right-handed-neutrinos—left-handed-antineutrinos, and make it really big so we don't see them at “low” energies.
2. Couple RH and LH neutrinos to Higgs.
3. Interplay gives LH neutrino mass matrix $m_{ij} \sim m^D_i\, m^D_j / M^{RH}$, where m^D is essentially the mass the neutrino would get from the Higgs mechanism without the explicit RH neutrino mass.
4. In case of multiple RH neutrinos, you have multiple M^{RH}, and also a matrix of Higgs couplings λ_{ij}.

The matter-antimatter asymmetry

- Taking m^D ~ “normal” masses and m~0.1 eV, we have $M^{RH} \sim 10^{14}$ GeV. (The GUT theorists think this is good.)
- Because the heavy neutrinos N_j are (hypothetically) so massive, they can decay into other particles.

$$N_j \rightarrow l^+ + \Phi^-$$
$$N_j \rightarrow l^- + \Phi^+$$

- The couplings λ_{ij} are allowed to have CP-violating phases, so these processes can have different rates.
- Another process in the standard model then turns this into baryon asymmetry.

This model is actually not so bad

- *CP violation* in *heavy neutrinos* leads to an imbalance of matter vs. antimatter in the very early universe, with only a little bit of “new physics”.
- This is called *leptogenesis*.
- I'm not aware of a viable model with a better “testable hypotheses to total hypotheses ratio.”
  - The biggest fly in the ointment is that CP violation in light neutrinos doesn't tell us everything about λ_{ij}, so we can't prove or disprove CP violation for the heavy neutrinos that way.

A modest research program to address the "leptogenesis" question

A) Measure θ_{13}. *[* We are here.]*

Forget step B if θ_{13} is too small.

B) Measure δ_{CP} of ordinary neutrinos.

C) Search for evidence of other neutrinos. *[*]*

D) Search for evidence that some are heavy.

E) Search for evidence of CP violation in heavy neutrinos, and/or unique signatures of this effect on early universe. [how to do this is unclear at present]

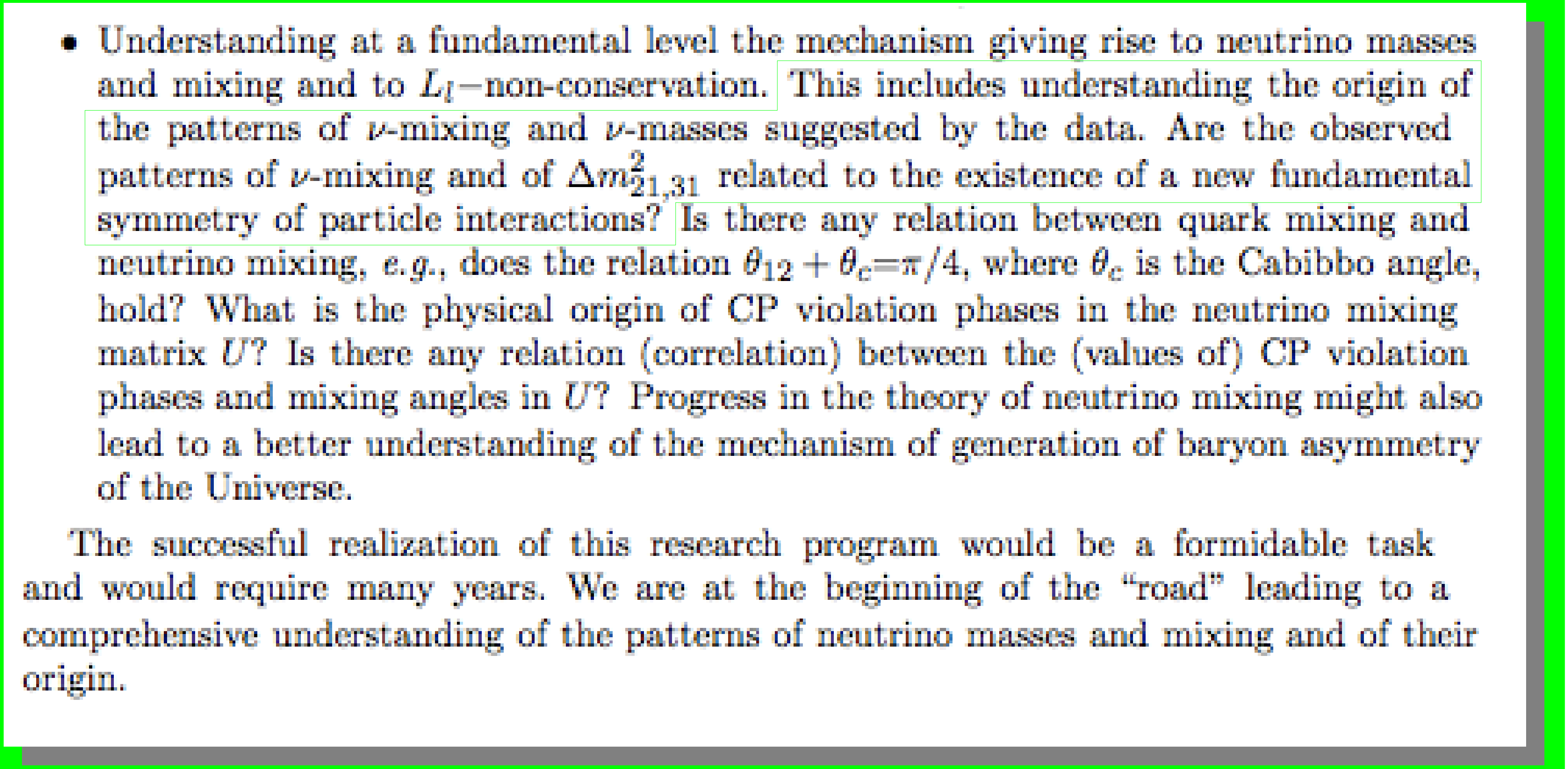
Near- and Mid-Term Outlook

- New reactor experiments to search for disappearance of electron antineutrinos: Double Chooz (now taking data), RENO, Daya Bay.
- Longer term accelerator experiments to search for electron neutrino appearance and make more precise measurements of other parameters and mass hierarchy.
- Neutrino-less double beta decay experiments also searching for evidence of Majorana nature of neutrinos (ν-$\bar{\nu}$ equivalence).

A far-range outlook from the neutrino chapter in the *Review*

- Determine Majorana or Dirac nature
- Determine mass hierarchy
- Determine absolute scale of mass
- Measure θ_{13} or improve limit by factor of 10.
- Determine CP symmetry broken/unbroken for ν.
- Measure all mixing parameters more precisely.
- “Understanding at a fundamental level the mechanism giving rise to neutrino masses and mixing and to L−non-conservation.”

More from the *Review* authors



- Understanding at a fundamental level the mechanism giving rise to neutrino masses and mixing and to L_l−non-conservation. This includes understanding the origin of the patterns of ν-mixing and ν-masses suggested by the data. Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new fundamental symmetry of particle interactions? Is there any relation between quark mixing and neutrino mixing, *e.g.*, does the relation $\theta_{12} + \theta_c = \pi/4$, where θ_c is the Cabibbo angle, hold? What is the physical origin of CP violation phases in the neutrino mixing matrix U? Is there any relation (correlation) between the (values of) CP violation phases and mixing angles in U? Progress in the theory of neutrino mixing might also lead to a better understanding of the mechanism of generation of baryon asymmetry of the Universe.

The successful realization of this research program would be a formidable task and would require many years. We are at the beginning of the "road" leading to a comprehensive understanding of the patterns of neutrino masses and mixing and of their origin.