Prospects for Experimental Access to the Neutrino Mass Hierarchy

Magic Baselines for Beam Neutrinos

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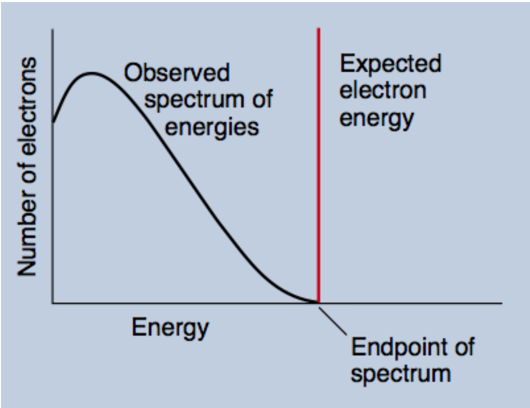
Overview

- Review of Neutrino history and physics
- Physics in Long Baseline Beam Experiments
- The "first" magic baseline
- The "short" magic baseline
- Conclusions

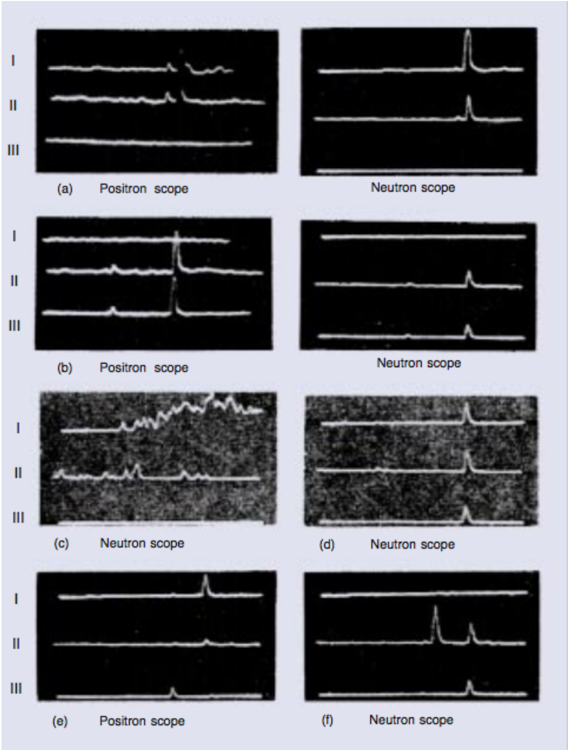
Neutrino Basics & History

Invented in 1930 by Wolfgang Pauli to solve a problem with β decay spectra



First observed in 1956 by Cowan and Reines using $\bar{\nu}$s from a nuclear reactor: $\bar{\nu} + p \rightarrow n + e^{+}$.

Only interact weakly.



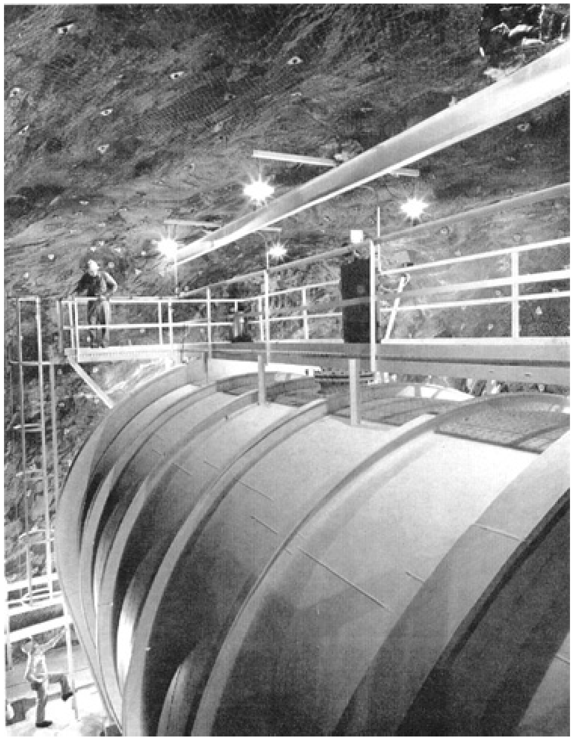
source: http://library.lanl.gov/cgi-bin/getfile?25-02.pdf

Into the Cuisinart

You ought to be abe to predict the neutrino flux from the sun ($\approx 6 \times 10^{14}/m^2/s$ at earth orbit), and with Cohen and Reines' results (i.e. the cross-section) you can design a counting experiment to measure it.

Raymond Davis Jr., Kenneth C. Hoffman and Don S. Harmer build a big tank in the Homestake mine and perform an *experimental tour de force* to measure it from 1970 to 1994.

It came up persistently up short of expectation. Very short.



Some are disappearing along the way. Where are they going??

source: http://www.sns.ias.edu/

Mixing

Neutrinos are produced in conjunction with charged leptons. Assume a set of lepton-flavor quantum numbers.

In that case Homestake only detects ν_e's (and *not* ν_μ's or ν_τ's). What if they are changing flavor in flight?*

This is only possible if the mass is non-zero. But mass is assumed to be zero. Therefore:

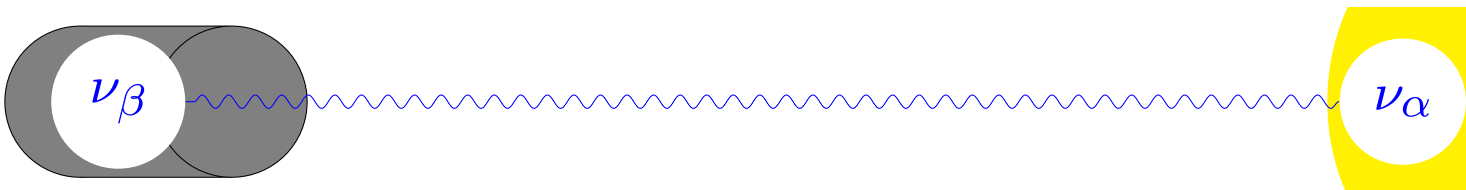
Physics beyond the standard model!

(*) Suggested by Gribov and Pontecorvo in 1968

jnb/Papers/Popular/Scientificamerican69/scientificamerican69.html

Formalism for Mixing

Three flavors: e, μ, τ	Three masses: m_1, m_2, m_3
Eigenstates of the weak interaction	Eigenstates of the free Hamiltonian



$$P(\nu_\alpha \to \nu_\beta) = \left|\langle \nu_\beta | \, \nu_\alpha(t)\rangle\right|^2 = \left|\left\langle \nu_\beta \left| U e^{-iEt} U^* \right| \nu_\alpha \right\rangle\right|^2$$

$$= \left| \sum_i \sum_j U^*_{\alpha,i} U_{\beta,j} \sin 2X_{i,j} \, \langle \nu_\beta | \, \nu_\alpha \rangle \right|^2$$

$$= \left| \sum_{j>i} \Re\left[U_{\alpha,i} U^*_{\beta,i} U^*_{\alpha,j} U_{\beta,j}\right] \sin^2 X_{i,j} + \sum_{j>i} \Im\left[U_{\alpha,i} U^*_{\beta,i} U^*_{\alpha,j} U_{\beta,j}\right] \sin 2X_{i,j} \right|^2$$

where U is a unitary mixing matrix and $X_{i,j} = \frac{m_i^2 - m_j^2}{4E} L$.

More Formalism

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$

Free Parameters

Three mixing angles: θ_{12}, θ_{23}, θ_{13}
A CP violating phase: δ_{CP}
(Two Majorana phases not shown: η_1, η_2)

Every term in the probability has at least *four* trig (θ_{ij})s in it!

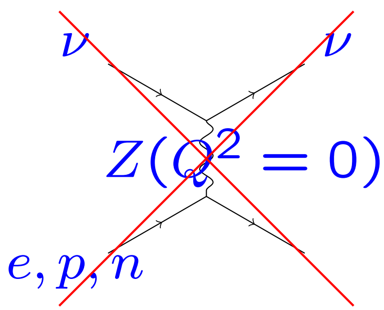
Knowns and Unknows

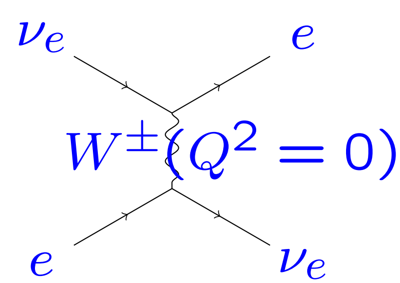
quantity	value	notes
$\sin^2(2\theta_{12})$	0.87 ± 0.03	KamLAND, solar, SNO, ...
$\sin^2(2\theta_{23})$	> 0.92	SuperK, MINOS...
$\sin^2(2\theta_{13})$	< 0.2	at 90% C.L. from Chooz
Δm^2_{21}	$7.6(2) \times 10^{-5}$ eV	KamLAND, solar, SNO ...
$\lvert\Delta m^2_{32}\rvert$ or $\lvert\Delta m^2_{31}\rvert$	$2.43(13) \times 10^{-3}$ eV	MINOS, atmospheric, K2K ...
δ_{CP}		Unknown

All $\theta_{i,j} \in [0, \frac{\pi}{2}]$ and $\delta_{CP} \in [0, 2\pi]$. Note that $\sin^2(2\theta_{13})$ and $\Delta m^2_{21}/\Delta m^2_{31}$ are "small".

Matter Effect

All of the above assumes the neutrino propagating in free space, but if the neutrino travels in matter the neutrinos can interact.





Hard scattering that removes the neutrinos from the beam are rare owning to the very small cross-section, but coherent forward scattering contributes and is different for electron neutrinos...

Matter Effect

Modification of the terms

The total effect of the coherent forward scattering is to add an effective potential $V = \sqrt{2} G_F n_e$ where n_e is the electron density.

The Hamlitonian there for looks like

$$\frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}U^* + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

where $A = 2VE$.

We write $\Delta m_{21}^2 = \alpha \Delta m_{31}^2$ and expand in terms of α.

Matter Effect

Simplification

That expansion lets you write the Hamiltonia as

$$\frac{\Delta m^2_{31}}{2E} U_{23} \left[U_{13} U_{12} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} U^*_{12} U^*_{13} + \begin{pmatrix} \frac{A}{\Delta m^2_{31}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] U^*_{23}$$

where the U_{ij}s are the sector by sector mixing matrices exhibited previously, and greatly reduces the number of terms to be retained.*

*Freund, Phys. Rev. D 64 053003 (2001)

Understanding long baseline physics

Consider a long baseline experiment either $P(\nu_e \rightarrow \nu_\mu)$ or $P(\nu_\mu \rightarrow \nu_e)$ with matter or anti-neutrinos.

$$P_{e\mu} \approx \sin^2 2\theta_{13} \sin^2 2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$
$$\pm \alpha \sin^2 2\theta_{13} \xi \sin \delta_{CP} \sin \Delta \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$
$$+ \alpha \sin^2 2\theta_{13} \xi \cos \delta_{CP} \cos \Delta \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$
$$+ \alpha^2 \cos^2 2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2} \quad (1)$$

where $\xi = \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$, $\hat{A} = \pm(2\sqrt{2}G_F n_e E)/\Delta m^2_{31}$, and $\Delta = \Delta m^2_{31} L/(4E)$

Positive(negative) for $\nu_e \rightarrow \nu_\mu$ ($\nu_\mu \rightarrow \nu_e$)

Same as (opposite of) Δm^2_{31} for neutrinos (anti-neutrinos)

The “first” magic baseline

If $\sin(\hat{A}\Delta)$ vanishes then equation 1 simplifies to

$$P_{e\mu} \approx \sin^2 2\theta_{13} \sin^2 2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

So set

$$\hat{A}\Delta = 2\sqrt{2} G_F n_e L/(4) = 2\pi$$

and solve for L in terms of an assumed constant value of n_e. You get 7630 km (or about 7250 if you use a numeric integration and a model Earth)

A long way!

Huber and Winter, Phys. Rev. D 68 (2003)

A Shorter Magic Baseline

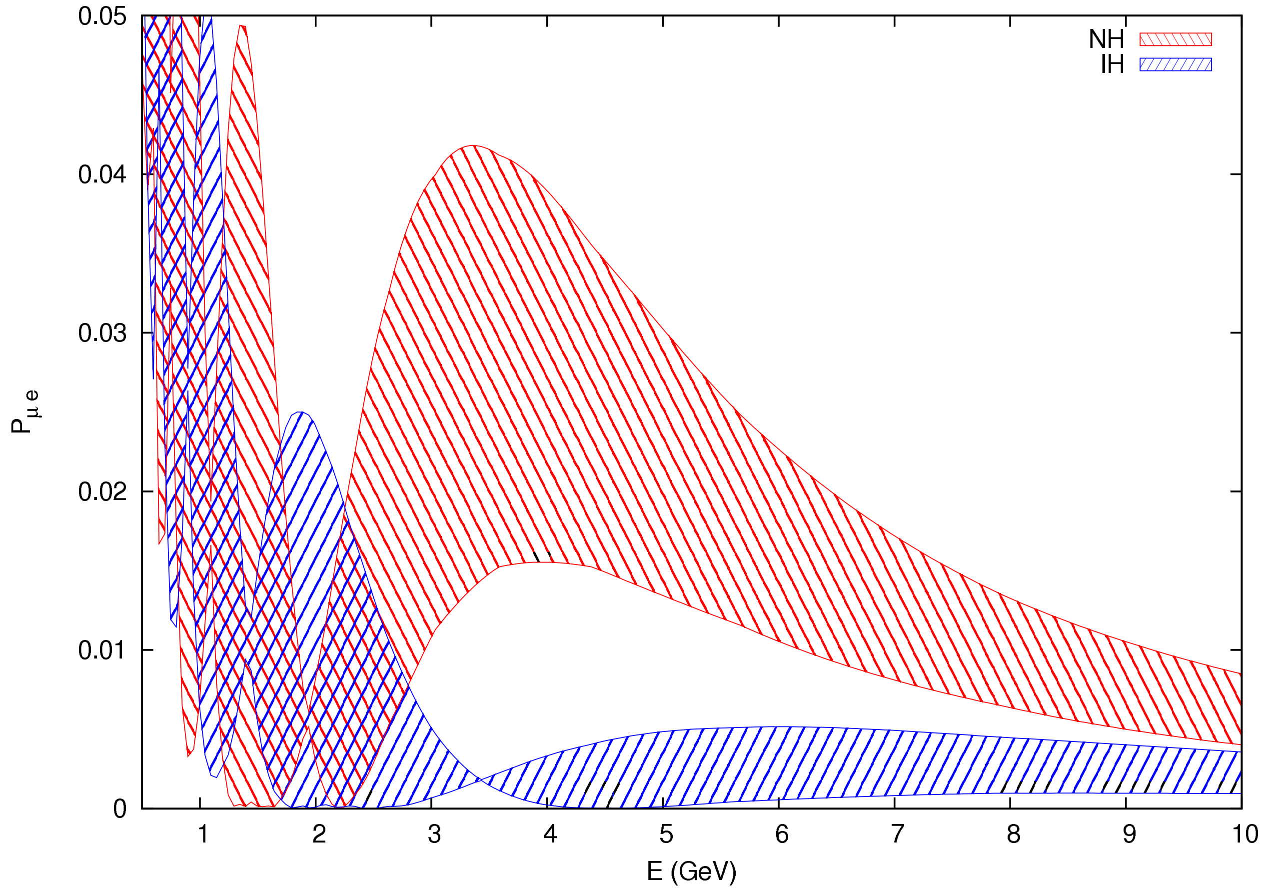
Choose L and E such that $\sin\left[(1-\hat{A})\Delta\right]$ cancels for the inverted hierarchy and is near maximal for the normal hierarchy.

$$(1-\hat{A})\Delta = \left(1 \mp \frac{2\sqrt{2}G_F n_e E}{\delta m^2_{31}}\right)\frac{\Delta m^2_{13} L}{4E}$$

$$= \frac{\Delta m^2_{31} L}{4E} \mp \frac{\sqrt{2}}{2} G_F n_e L = \begin{cases} \pi & \text{Inverted hierarchy} \\ \pi/2 & \text{Normal hierarchy} \end{cases}$$

Solved this gives $L = 2540$ km and $E = 3.3$ GeV.

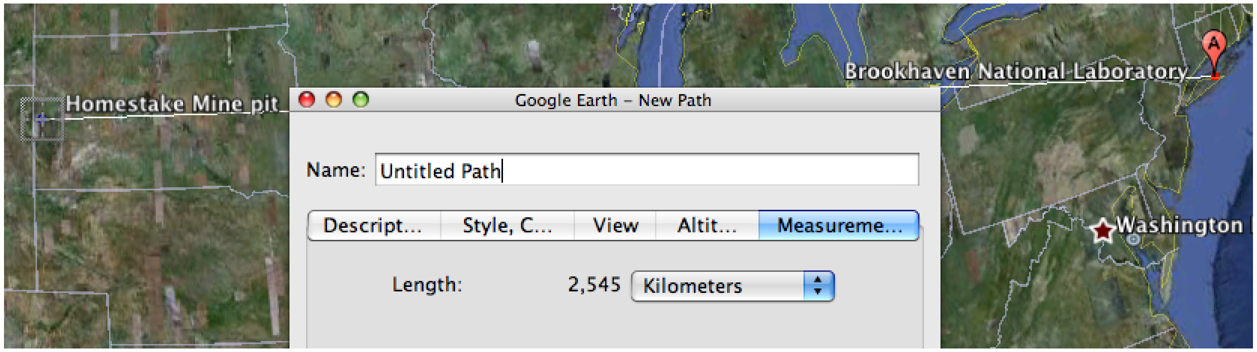
Expectations at the Short Magic Baseline



arXiv 0908:3741v3

Hey, Look!

In terms of detector rate, 2500 km is a lot more manageable than 7500 km and it so happens that we have a plausible accelerator/detector site pair...

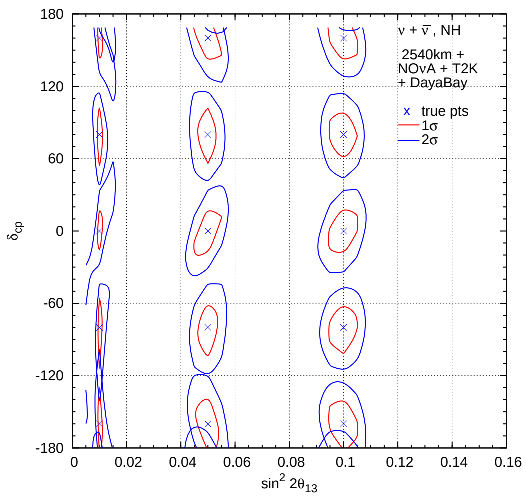


Sensitivity

$\sin^2 2\theta_{13}$	Exposure (kton-years) Normal	Inverted
0.10	0.022	0.048
0.08	0.026	0.068
0.06	0.051	0.105
0.04	0.104	0.195
0.02	0.425	2.600

The table shows the estimated exposure needed to make a 3σ determination of the mass hierarchy as a function of the true value of θ_{13} and assuming a 10^{23} POT per year at 2540 km and assuming neutrino only running.

Because the beam is not mono-energetic there *is* sensitivity to δ_{CP}. The figure shows expected error bars around 15 pairs of $(\theta_{13}, \delta_{CP})$ including systematics for 5 years neutrinos and 5 years anti-neutrinos assume a normal hierarchy.



Conclusion

- Magic baselines reduce the ambiguity implicit in 3-flavor oscillation (especially with the matter effect)

- The first (long) magic baseline is another handle on θ_{13} independent of energy and CP violating effects. There is the potential for high precision than from existing reactor experiments. But sill suffer from low far detector rates for the foreseeable future.

- The "short" magic baseline is more practical and is primarily sensitive to the mass hierarchy.