



$\sin^2 \theta_{13}$ VALUE LIMIT IN
DOUBLE CHOOZ EXPERIMENT
Pi-Jung Chang

OUTLINE

- Discovery neutrino
- Neutrino oscillation
- Double Chooz experiment
 - Detector structure
 - Neutrino source
 - Backgrounds
- Systemic uncertainties
 - Uncertainties in Double Chooz experiment
 - Sensitivity models
- Recent work
 - Energy scale
 - Environmental monitoring system
 - Final fitting process
- Conclusion



DISCOVERY NEUTRINO

- The neutrino was first postulated theoretically.
 - Pauli -> Beta decay in 1930
- $$M(A, Z) \rightarrow D(A, Z + 1) + e^- + \bar{\nu}_e$$
- prove by Cowan and Reines(1956)
 - Electron anti-neutrino from nuclear reactors
$$\bar{\nu}_e + p \rightarrow e^+ + n$$
- 1962 muon neutrinos (Danby et al 1962)
- 1975 tau lepton-> tau neutrinos(Kodama et al 2001)

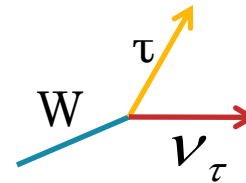
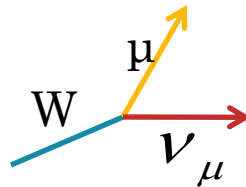
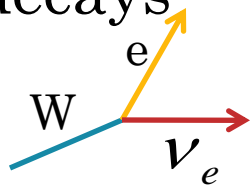
Three Generations of Matter (Fermions)

	I	II	III	
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name →	u up	c charm	t top	γ photon
	4.8 MeV	104 MeV	4.2 GeV	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	d down	s strange	b bottom	g gluon
	<2.2 eV	<0.17 MeV	<15.5 MeV	91.2 GeV
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z weak force
	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV
	-1	-1	-1	±1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	e electron	μ muon	τ tau	W[±] weak force



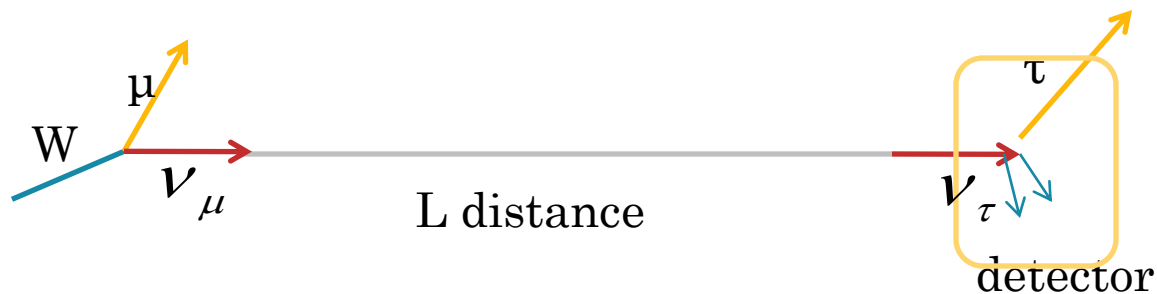
NEUTRINO OSCILLATION

- Three known flavor neutrinos from W boson decays



- Neutrino Flavor Change

- If neutrino has masses and leptons mix.



NEUTRINO OSCILLATION

- Linear superpositions of mass eigenstates

$$|\nu_l\rangle = \sum_i U_{l,i} |\nu_i\rangle \quad l=e,\tau,\mu ; i=3,4,\dots$$

Weak eigenstates $|\nu_l\rangle$, mass eigenstates $|\nu_i\rangle$, leptonic mixing matrix $U_{l,i}$, at least $i=3$ mass eigenstates.

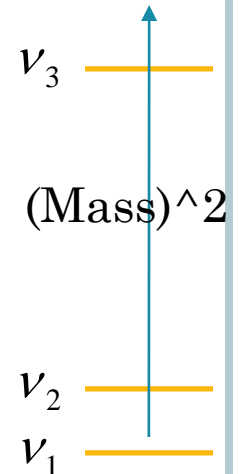
$$|\nu_i(t)\rangle = e^{-i(E_i t - pL)} |\nu_i(0)\rangle \cong e^{-i(m_i^2/2E)L} |\nu_i(0)\rangle$$

$$|\nu_l(L)\rangle \cong \sum_i U_{l,i} e^{-i(m_i^2/2E)L} |\nu_i(0)\rangle \cong \sum_l \sum_i U_{l,i} e^{-i(m_i^2/2E)L} U_{l',i}^* |\nu_{l'}(0)\rangle$$

- Probability

$$P(\nu_l^{(-)}(L) \rightarrow \nu_{l'}^{(-)}) = \left| \sum_i U_{l,i} U_{l',i}^* e^{-i(m_i^2/2E)L} \right|^2 = \delta_{\alpha\beta} - 4 \sum_{i>j=1}^3 \text{Re}(K_{\alpha\beta,ij}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + 4 \sum_{i>j=1}^3 \text{Im}(K_{\alpha\beta,ij}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \cos\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$

$$K_{\alpha\beta,ij} = U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}$$



NEUTRINO OSCILLATION

○ The Mixing Matrix U

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Atmospheric Cross-Mixing Solar

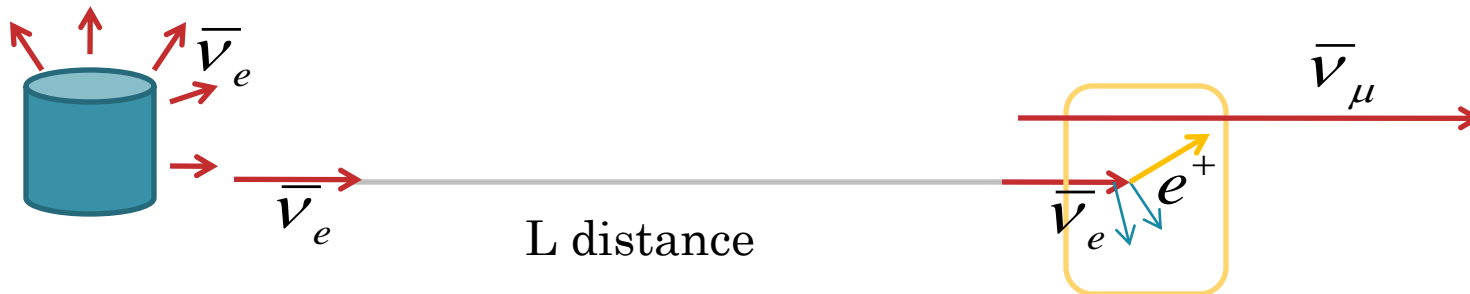
$$c_{ij} \equiv \cos \theta_{ij}, s_{ij} \equiv \sin \theta_{ij}, \theta_{12} \sim \theta_{sol}, \theta_{23} \sim \theta_{atm}.$$

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$ CP violation

- CP violation disappears : $\sin \theta_{13} \rightarrow 0$

REACTOR NEUTRINO EXPERIMENTS PROPERTIES

- $\bar{\nu}_e$ -> beta decay of the neutron-rich fission fragment
- Energy is low (few MeV) -> only $\bar{\nu}_e$ disappearance
- Flux -> large solid angle and low signal rates
- There is no beam alignment problem



“Reactor-based Neutrino Oscillation Experiment” Carlo
Bemporad et al..

“Neutrino Physics” Kai Zuber.



REACTOR EXPERIMENTS

○ Reactor neutrino experiment

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} = 1 - \cos^4 \theta_{13} \sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) - \sin^2 2\theta_{13} \sin^2 \theta_{12} \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

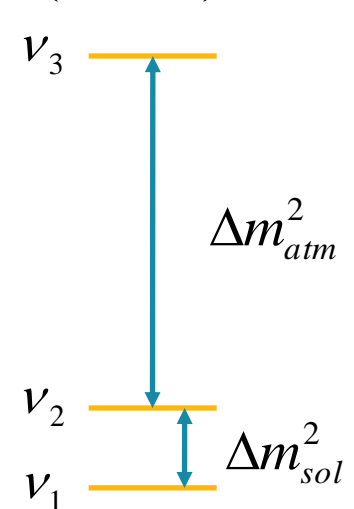
$$- \sin^2 2\theta_{13} \cos^2 \theta_{12} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$\cong 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

- Doesn't depend on the δ -CP phase
- Short baseline experiment
- Experimental data

$$\Delta m_{atm}^2 \cong 2.4 \times 10^{-3} eV^2, \Delta m_{sol}^2 \cong 7.6 \times 10^{-5} eV^2$$

$$\sin^2 2\theta_{23} \cong 1, \sin^2 2\theta_{12} \cong 0.8, \sin^2 2\theta_{13} < 0.1 \quad \text{Chooz data}$$

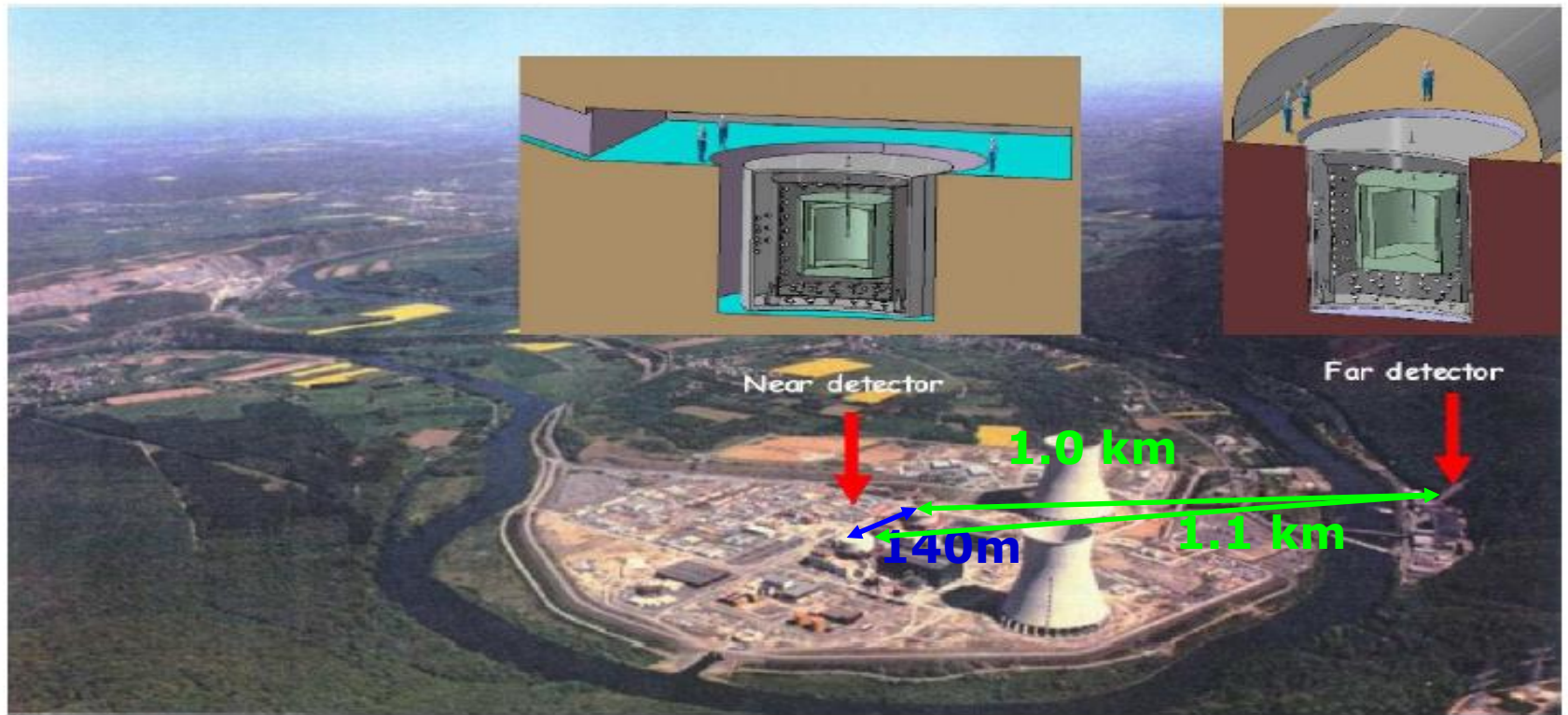
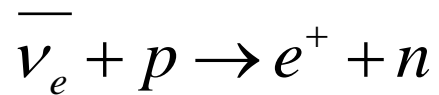


LOCATION

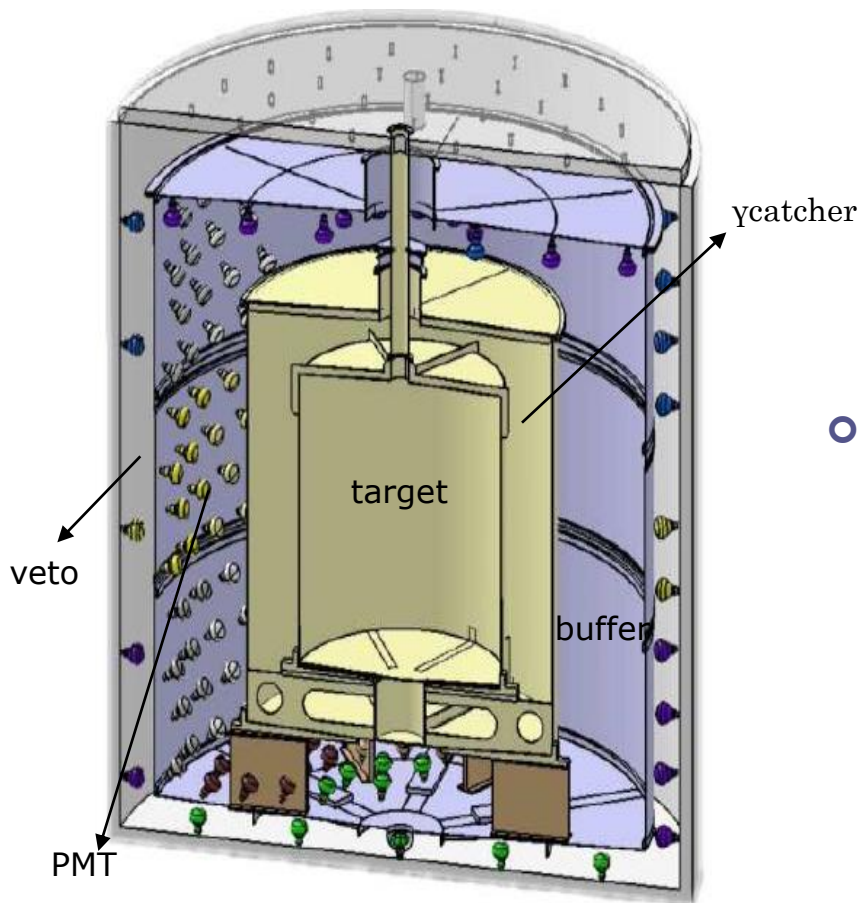


Introduction Double CHOOZ

- Antineutrino flux \leftarrow the two nuclear cores of the Chooz power plant results from β^- decay of the fission products of four main isotopes ^{235}U , ^{239}Pu , ^{241}Pu and ^{238}U .

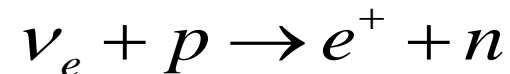


Detector structure



- Scintillator (target):

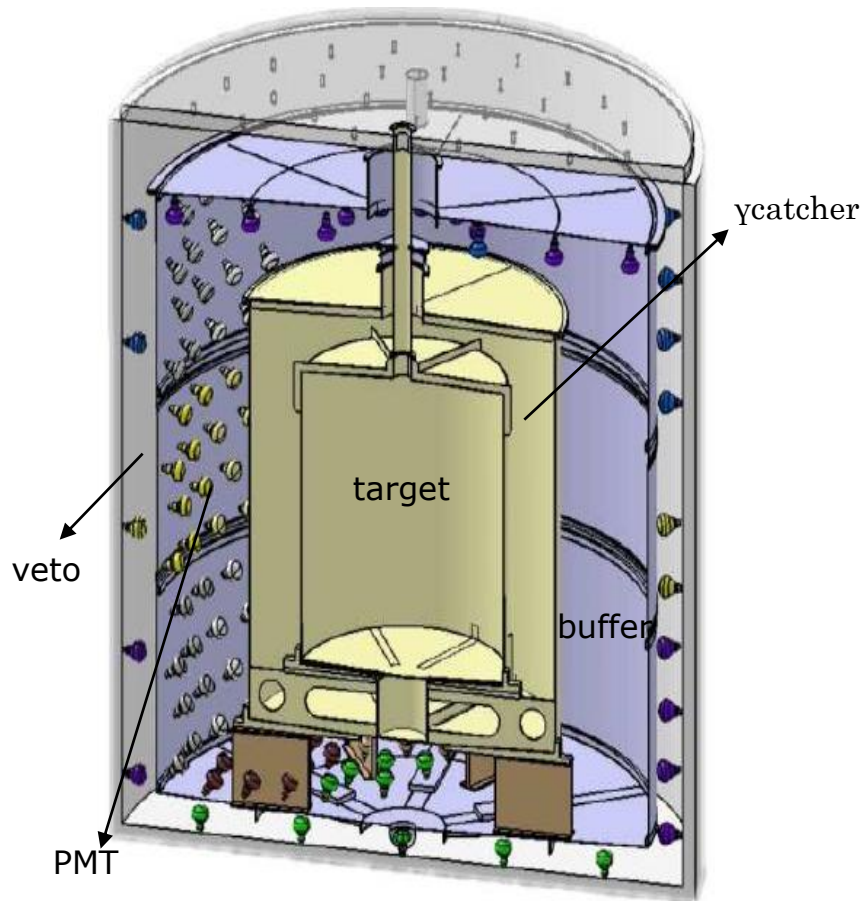
- Proton \rightarrow 20% PXE(C₁₆H₁₈) , 80% of dodecane(C₁₂H₂₆), small amount gadolinium
- Neutron capture \rightarrow small amount gadolinium



- Γ catcher:

- the same optical properties as target.
- get the full positron annihilation energy.
- most of the neutron energy released after neutron capture.

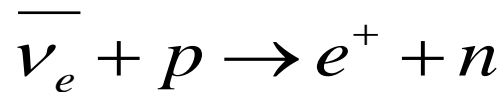
Detector structure



- Buffer (stainless steel and PMT support, mineral oil):
 - reduce the single rate in target and gamma catcher.
 - Lower the positron threshold down to 500keV
- Inner veto (mineral oil): muon tagging and fast neutron background rejection.
- achieve a light yield about 200 pe/MeV
- Total PMT number 468 for each.



EXPERIMENTAL METHOD

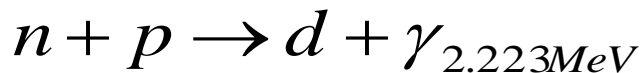


- Positron : deposits kinetic energy -> scintillation light and annihilation.

$$E_{\bar{\nu}_e} \cong 1.293(\text{MeV}) + E_{e^+}$$

- Neutrino

- Neutron captures on hydrogen



- Neutron capture on Gd

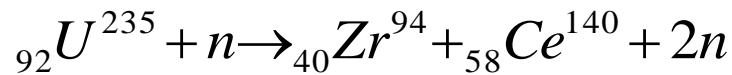


- Two energy need to be correlated in time and space->neutrino events



ANTINEUTRINO EVENTS PREDICTIONS

- $\bar{\nu}_e$ Spectrum from source β^- decay



- Electron spectra associated with the thermal neutron $\rightarrow \bar{\nu}_e$ spectra.

$$\frac{dN}{dE_e} \approx \frac{dN}{dE_{\bar{\nu}_e}}$$

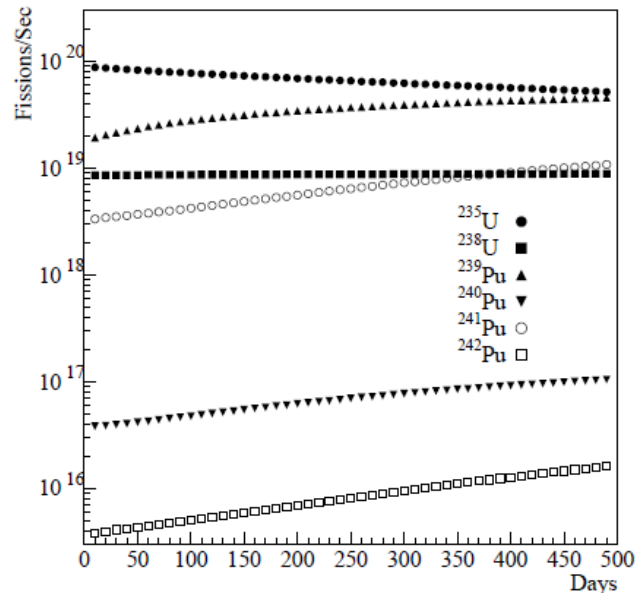


FIG. 6. Time evolution of fission rates for each of the six most important isotopes in one of the Palo Verde reactor cores. The horizontal scale covers a full fuel cycle, at the end of which about 1/3 of the core is replaced with fresh fuel. Only the four most important isotopes are normally used to predict $\bar{\nu}_e$ yields.

“Reactor-based Neutrino Oscillation Experiments“ Carlo Bemporad et al.



ANTINEUTRINO EVENTS PREDICTIONS(NO OSCILLATION)

- Antineutrino cross-section on proton

$$\langle \sigma \rangle_{fission} \cong 5.825 \times 10^{-43} \text{ cm}^2$$

- Number of fission per second($P_{th} \sim 4.27 \text{ GW}_{th}$, $W \sim 203.87 \text{ MeV}$)
$$N_f = 6.241 \times 10^{18} \text{ sec}^{-1} \times (P_{th} [MW] / W [MeV])$$
- Antineutrino events rate

$$R_L = \frac{N_f \times \langle \sigma \rangle_{fission} \times n_p}{4\pi L^2} \cong 60 \text{ events / day}$$



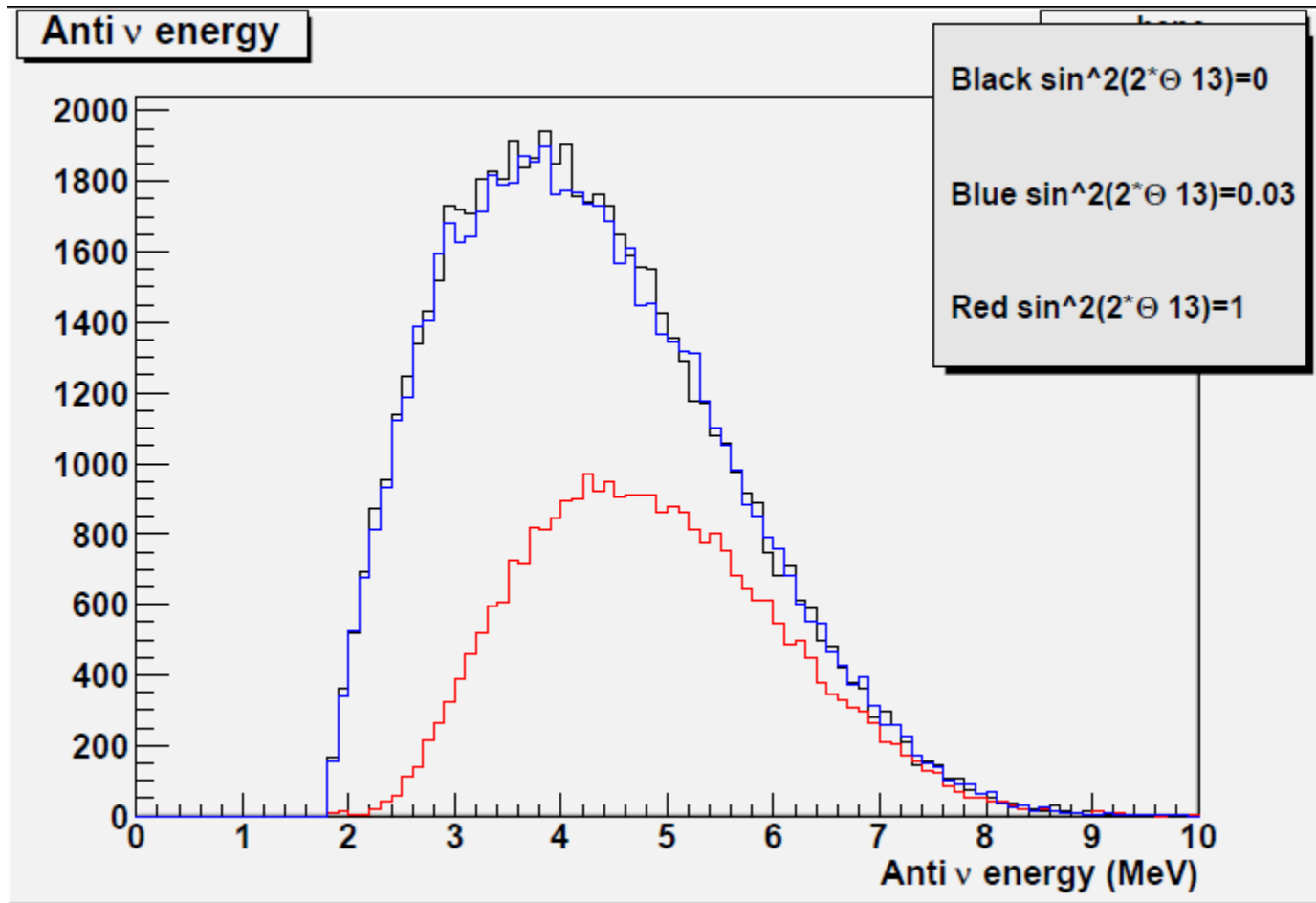
ANTINEUTRINO EVENTS PREDICTIONS(OSCILLATION)

$$N_i = F \int_{E_i}^{E_{i+1}} \int_0^{+\infty} S(E_\nu, E'_\nu) \sigma(E_\nu) \phi_i(E_\nu, L) P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(E_\nu, L) dE_\nu dE'_\nu$$

- F: normalization factor
- S: energy resolution effect
- $\sigma(E_{e^+}) \cong \frac{2\pi^2 \hbar^3}{m_e^5 f \tau_n} p_{e^+} E_{e^+}$, cross section for inverse β -decay
- ϕ : antineutrino flux
- P: antineutrino flux survival probability



1000DAYS MC NEUTRINO SPECTRUM



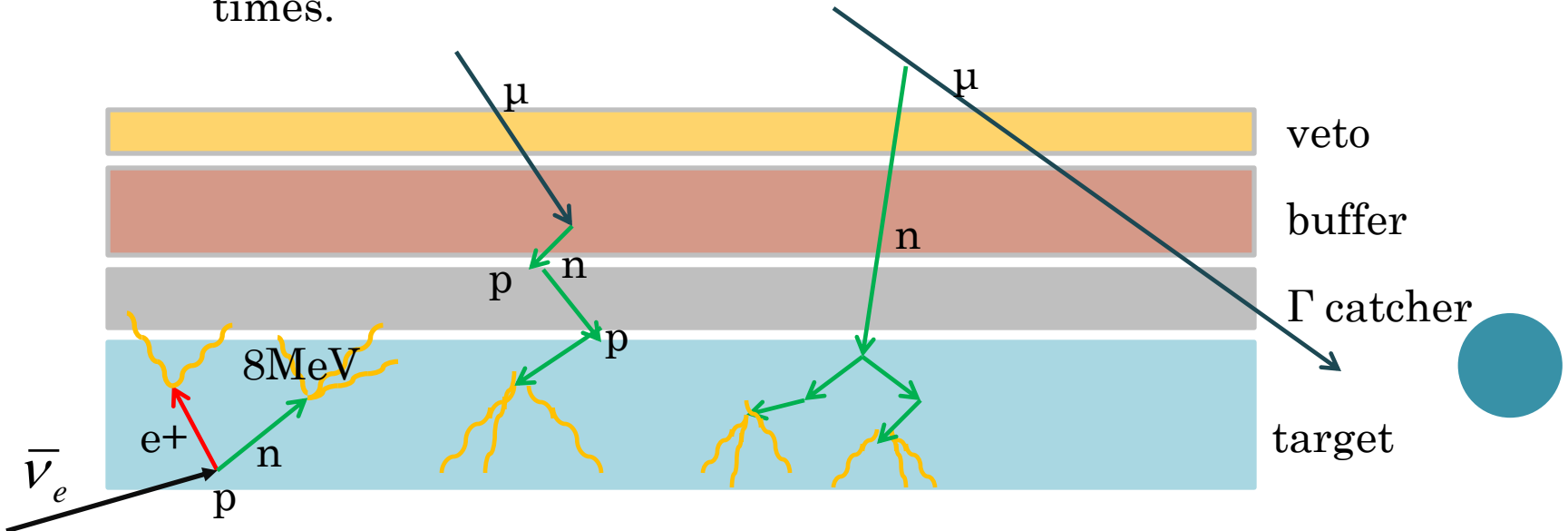
BACKGROUNDS

- $\bar{\nu}_e$ Interaction deposits at least 1 MeV and 8 MeV neutron capture in gadolinium.
- Background -> neutron like events
 - Into time window (few 100 μ s)
 - Over 1 MeV
- Beta and gamma background
 - U, Th and K from scintillator mass, acrylic vessels, photomultiplier and structure material. (>1 MeV)
 - ^{208}Tl in buffer region 2.6 MeV gamma emission.



NEUTRON BACKGROUND: NEUTRON LIKE EVENTS

- External cosmic muons
 - Through going muons \rightarrow spallation process
 - Estimated by muon fluxes, mean energies and shielding factors
 - Stopped negative muons \rightarrow captured by nuclei in target
 - Estimated by depth of shielding, μ -life time and μ -capture times.



○ Beta-neutron cascades

- Muon spallation on $^{12}\text{C} \rightarrow {}^8\text{He}, {}^9\text{Li}, {}^{11}\text{Li}$ undergo beta decay with a neutron emission
- Uncorrelated muon events: one event per day for far and 9~23 events per day for near.



EXPERIMENT SYSTEMATIC UNCERTAINTIES

- Main errors of CHOOZ
 - Antineutrino flux and spectrum(1.9%)
 - Cross section of neutrinos on the target protons.(0.8% number of protons, detector efficiency 1.5%)
- CHOOZ total error 2.7%→Double CHOOZ reduces to 0.6%.
 - reduce the systematic errors and background.
 - Two identical detectors →negligible reactor flux and cross section of neutrinos on the target protons.



ANALYSIS CUTS TO SELECT THE ANTINEUTRINO

- 7 analysis cuts (CHOOZ)-> 3 analysis cuts
 - Reduce analysis cuts -> reduce uncertainty
- Prompt positron signal
 - Energy cut at 500keV and 200 μ s.(Antineutrino interaction least 1MeV)
- Neutron delayed signal, cut at 6MeV
 - Neutron capture on hydrogen at 2.2MeV
 - Neutron capture on gadolinium at 8MeV
- neutron capture on gadolinium is less than 200 μ s.
- Prompt and delay event distance.(2m)



OTHER UNCERTAINTIES

- Solid angle
 - Distance between reactors and detectors.(keep below 0.2%)
- Uncertainty on the scintillator density
 - We need to know the mass of the target to $<0.2\%$.
- Neutron efficiency(captured by H and Gd)

	CHOOZ	Double CHOOZ
Reactor power	0.7%	Negligible
Energy per fission	0.6%	Negligible
Antineutrino/Fission	0.2%	Negligible
Neutrino cross section	0.1%	Negligible
Number of protons/cm ³	0.8%	0.2%
Neutron time capture	0.4%	Negligible
Neutron efficiency	0.85%	0.2%
Neutron energy cut	0.4%	0.2%



SENSITIVITY

- Least squares Minimization

$$m_i = t_i(\bar{p}) + r_i \sigma_i + \sum_k s_k \Delta_{ik}$$

- m: measurement of data-background, t: the model prediction, σ : the uncorrelated (statistical) error, Δ : correlated (systematic error) from source k.

- Probability density function of measurements

- $$P = C \exp\left(-\frac{1}{2} \chi^2\right), \chi^2 = \sum_{ij} \left(\frac{m_i}{t_i} - 1\right) V_{ij}^{-1} \left(\frac{m_j}{t_j} - 1\right), V_{ij} = \delta_{ij} \sigma_i^2 + \sum_k \Delta_{ik} \Delta_{jk}$$

- Correlated : theoretical cross section of detector, reactor fluxes(spectrum of antineutrino flux)
- Uncorrelated: proton numbers, baseline lengths, a part of detector efficiency, background..



SENSITIVITY MODELS

- one detector one reactors model

$$\chi^2 = \left(\frac{m}{t} - 1\right) V^{-1} \left(\frac{m}{t} - 1\right) = \frac{[(m/t - 1)]^2}{\sigma_c^2 + (\sigma_u^{(r)})^2 + (\sigma_c^{(r)})^2 + \sigma_u^2}$$

$$\cong \frac{\sin^4 2\theta \left\langle \left\langle \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \right\rangle \right\rangle^2}{\sigma_c^2 + (\sigma_u^{(r)})^2 + (\sigma_c^{(r)})^2 + \sigma_u^2}, V = \sigma_c^2 + (\sigma_u^{(r)})^2 + (\sigma_c^{(r)})^2 + \sigma_u^2$$

$$\left\langle \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \right\rangle = F \int_{E_i}^{E_{i+1}} \int_0^{+\infty} S(E_\nu, E'_\nu) \sigma(E_\nu) \phi_i(E_\nu, L) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) dE_\nu dE'_\nu$$

Form : “Systematic limit on $\sin^2 2\theta_{13}$ in neutrino oscillation experiments with multi-reactor” H. Sugiyama et al.



SENSITIVITY MODELS

- two detector one reactor model

$$\chi^2 = \left(\frac{m_n}{t_n} - 1, \frac{m_f}{t_f} - 1 \right) V^{-1} \begin{pmatrix} \frac{m_n}{t_n} - 1 \\ \frac{m_f}{t_f} - 1 \end{pmatrix} = \frac{[(m_n/t_n - 1) + (m_f/t_f - 1)]^2}{4\sigma_c^2 + 4(\sigma_u^{(r)})^2 + 4(\sigma_c^{(r)})^2 + 2\sigma_u^2} + \frac{[(m_n/t_n - 1) - (m_f/t_f - 1)]^2}{2\sigma_u^2}$$

$$\cong \sin^4 2\theta \left[\left(\left\langle \sin^2 \left(\frac{\Delta m^2 L_f}{4E} \right) \right\rangle - \left\langle \sin^2 \left(\frac{\Delta m^2 L_n}{4E} \right) \right\rangle \right)^2 / 2\sigma_u^2 \right]$$

Form : “Systematic limit on $\sin^2 2\theta_{13}$ in neutrino oscillation experiments with multi-reactor” H. Sugiyama et al.

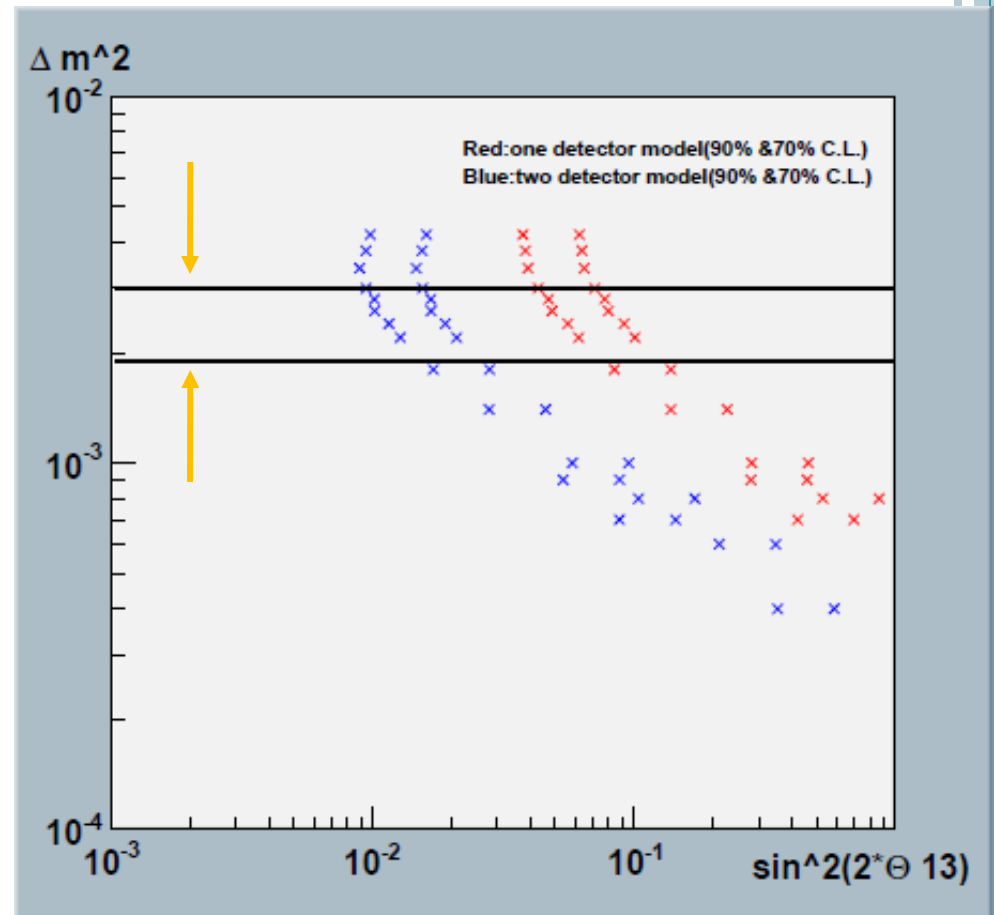


SENSITIVITY OF $\sin^2 2\theta_{13}$ (ONE DETECTOR VS. TWO DETECTOR ONE REACTOR MODEL)

○ 100days MC data,

$$\Delta m_{31}^2 = 2.4_{-0.5}^{0.6} \times 10^{-3} eV$$

	90% C.L.	70% C.L.
Two detector	0.02~0.015	0.0127~0.009
One detector	0.10~0.07	0.06~0.04



Recent work: Energy scale

- To reconstruct the conversion function between a measurable detector quantity and energy.
- processes
 - The energy of measured particles
 - Quenching of the scintillation light
 - Cerenkov light

$$\frac{E_{visible}}{E_{real \ \gamma}}(E) = S [Q_{\gamma}(K_b, E) + C_{\nu} P_{\gamma}(E)]$$

“A precise determination of the Kamland energy scale” Timothy M. Classen thesis.



Scintillator

○ Solvents

- 20% PXE($C_{16}H_{18}$): collect energy, conduct energy to fluor.
- 80% of dodecane($C_{12}H_{26}$): improve the chemical compatibility with the acrylic, increase the number of free protons.

○ Fluor (0.3~1%)

- Primary-> PPO(emission λ 357 nm): able to be excited to a light emitting state by excited solvent molecules.
- Secondary-> Bis-MSB(emission λ 420 nm): wavelength shifter.

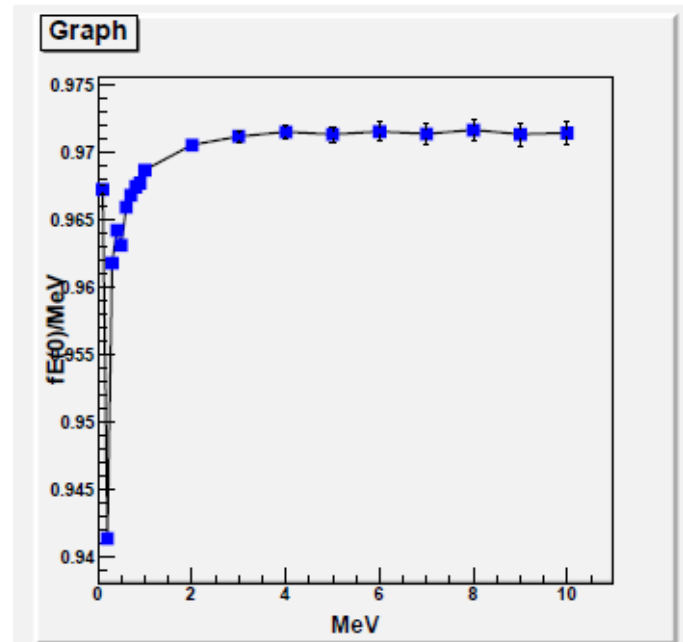


IONIZATION QUENCHING

- When a large amount of energy is deposited in a small area, some of molecules in scintillator will shed their excess energy thermally through a quenched energy.
- Birks' Law

$$\frac{E_{visible}}{E_{real}} = \frac{S}{E_{real}} \int_0^E \frac{dE}{1 + K_b \frac{dE}{dx}}$$

- K_b is Birks' constant, S scintillation efficiency, E is particle energy, and x is the range in scintillator.

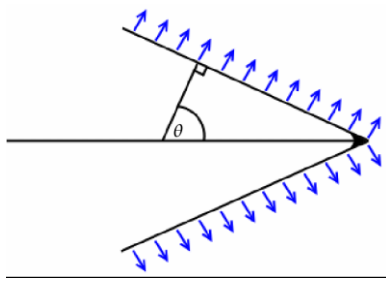


Simulation by MC

“A precise determination of the Kamland energy scale” Timothy M. Classen thesis.

Cerenkov light

- Cerenkov radiation is produced whenever a charged particle is traveling faster than the local speed of light.



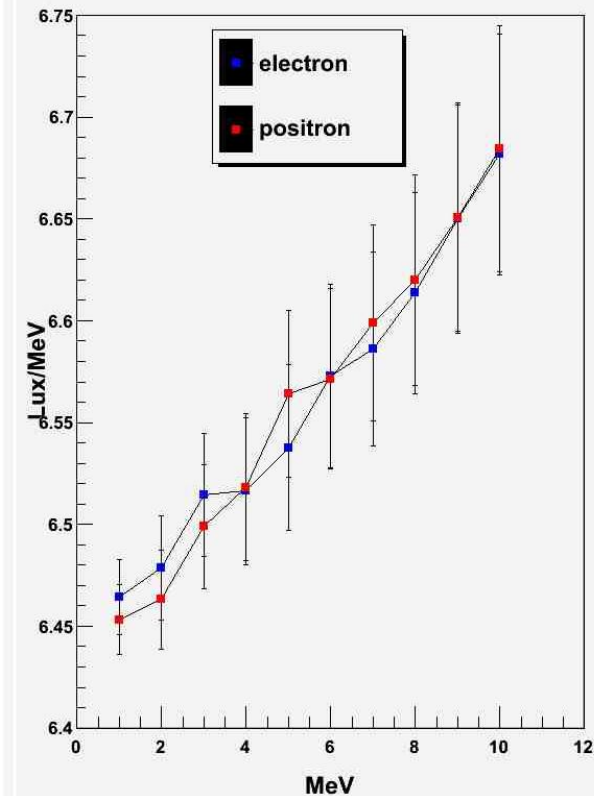
$$\frac{d^2 N}{dx d\lambda} = \frac{2\pi\alpha z^2}{\lambda^2} \left(1 - \frac{1}{\beta^2 n(\lambda)^2} \right)$$

- N is number of photons, x is track length, α is the fine structure constant and z is the particle's charge.
- The light will absorb by PXE and reemit by fluor

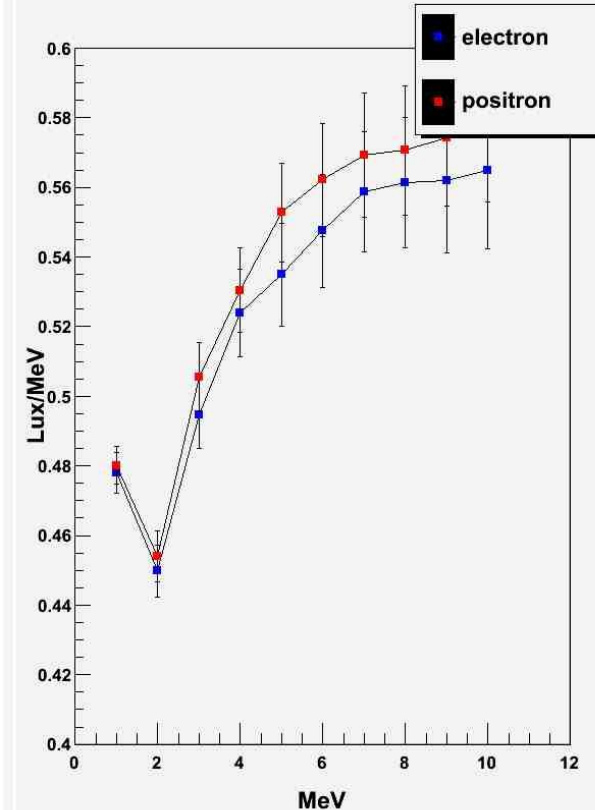


QUENCH AND CERENKOV LIGHT FLUX IN DETECTOR

Only Quench light



Only Cerenkov light



ENERGY SCALE WORK

○ Quench

- the quenching bench measurements->Values of Birk's parameter K_b for MC .
- Calibration source natural alpha sources-> produce no Cerenkov light.
- Compare calibration data->adjust K_b constants.

○ Cerenkov light

- Calibrated by Compton scattering



RECENT WORK: ENVIRONMENTAL MONITORING

- “Slowmon system”, environmental monitoring system installation is done in far detector.
 - environmental monitoring system, Control systematic effects and Provide alarms, warnings, diagnostics
 - Detected items: temperature, magnetic field around PMT, voltage values of Front-end boards and hall and control room temperature and humidity.



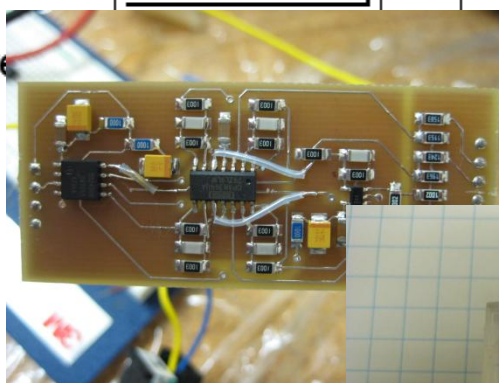
IP network



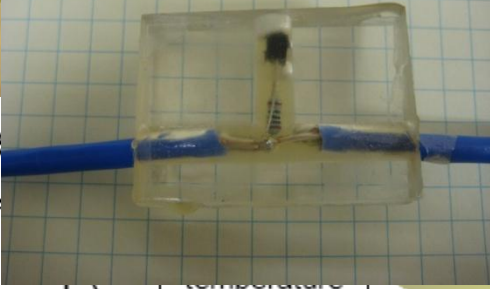
data sources



NIM crate



NIM crate



temperature monitor

(5 crates, ~60 cards)

MySQL server

FEE 1-Wire

veto

buffer

Switch box

box

box

DS9490

Hall 1-Wire net

AC power monitor

temperature and humidity monitor

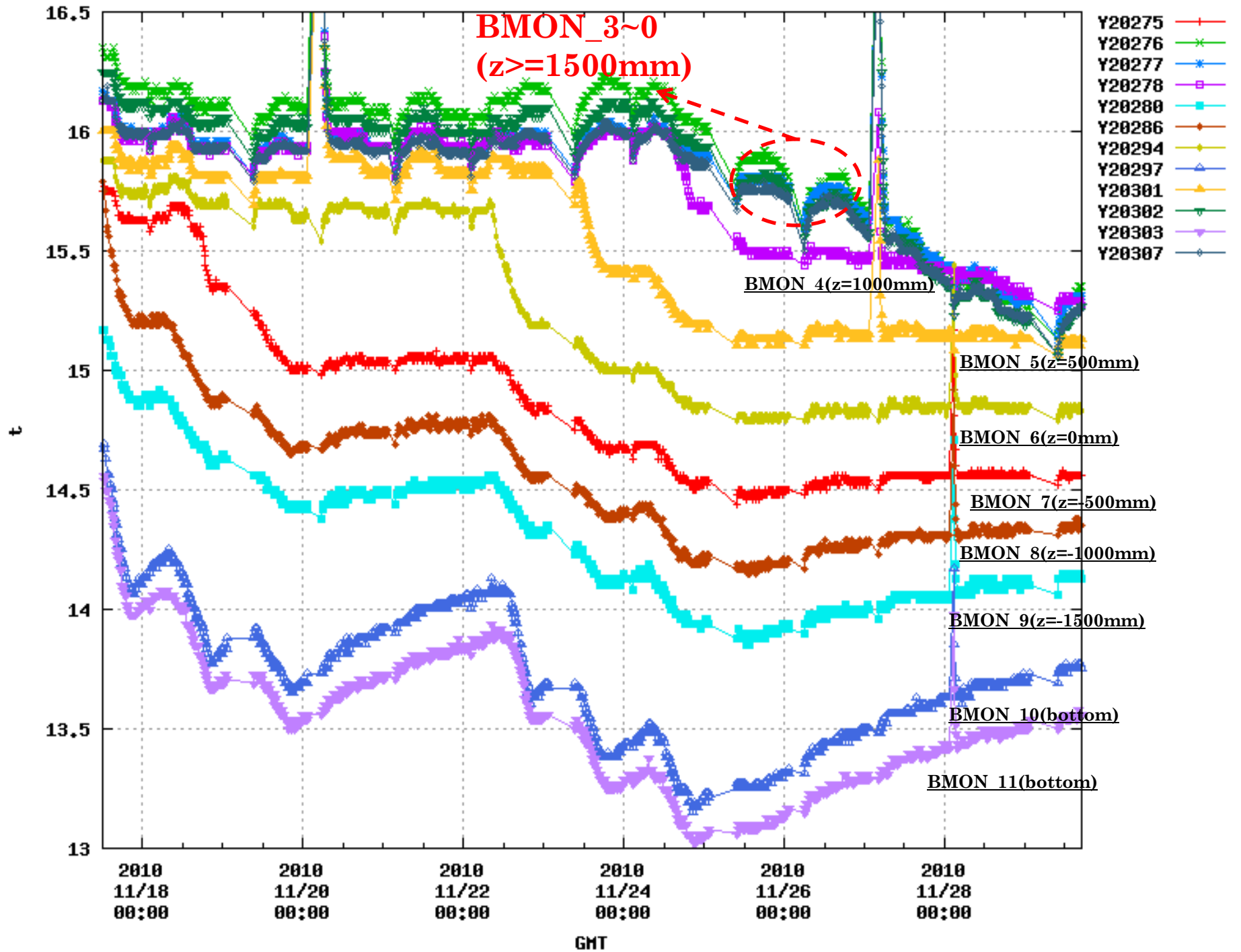
g pu AD

(other hall environment monitors)



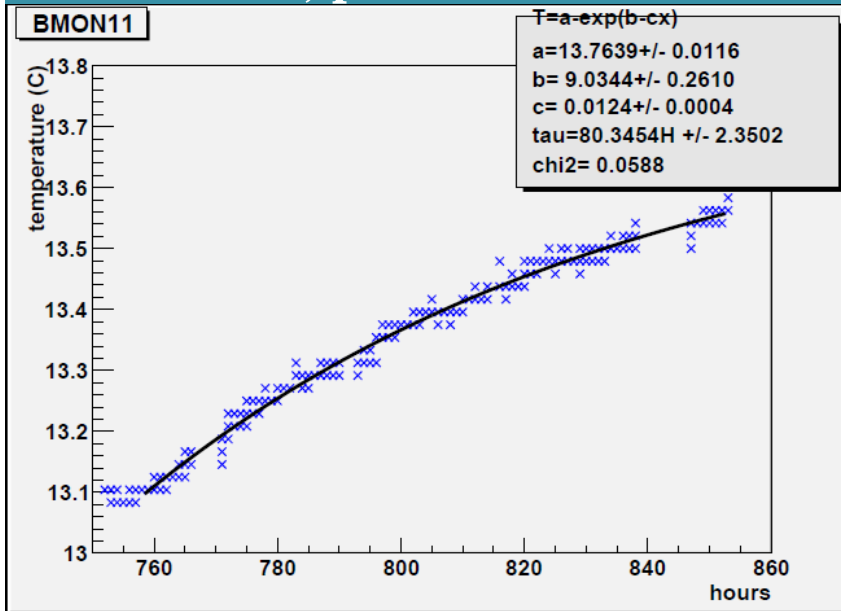
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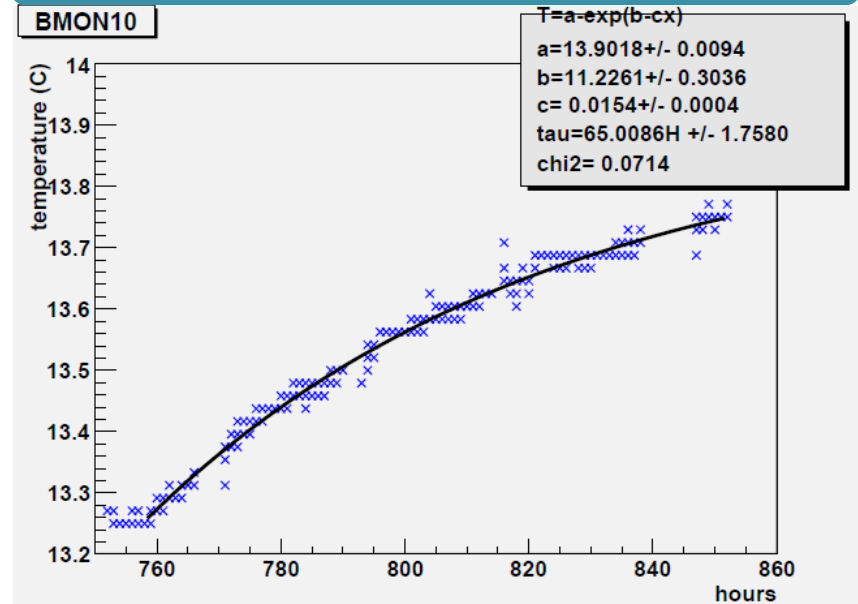


Buffer tank thermal meters data from 11/25~11/29

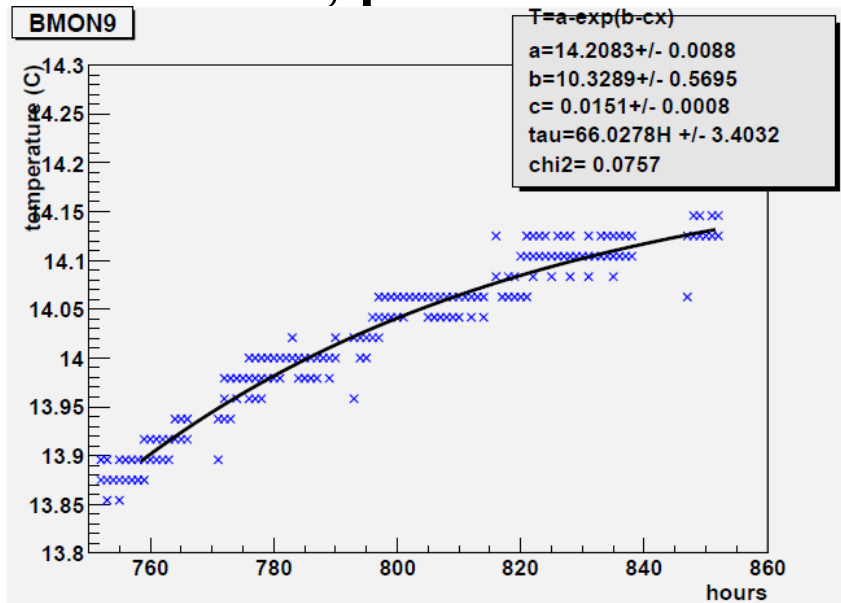
-2831.4mm, phi141.6 Buffer



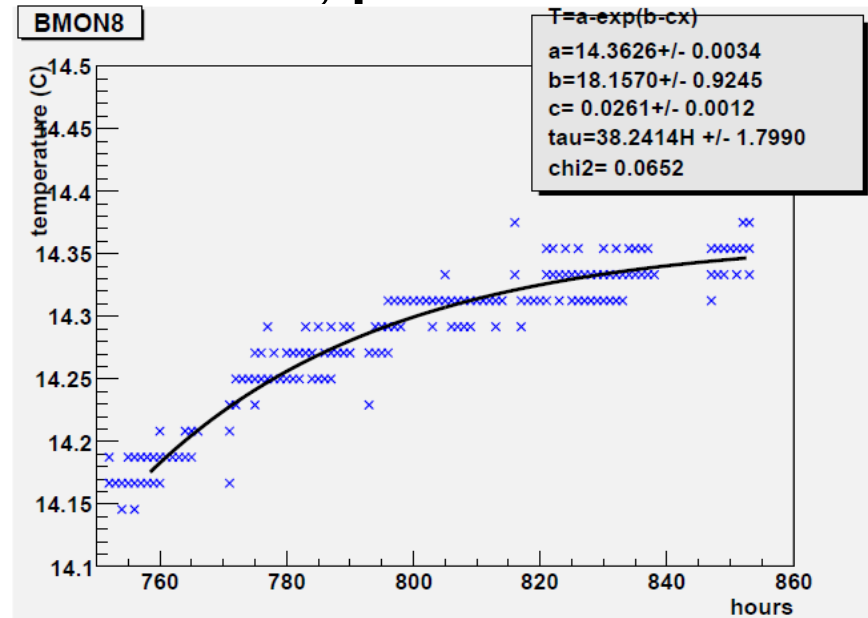
-2804.5mm, phi 86.3 Buffer



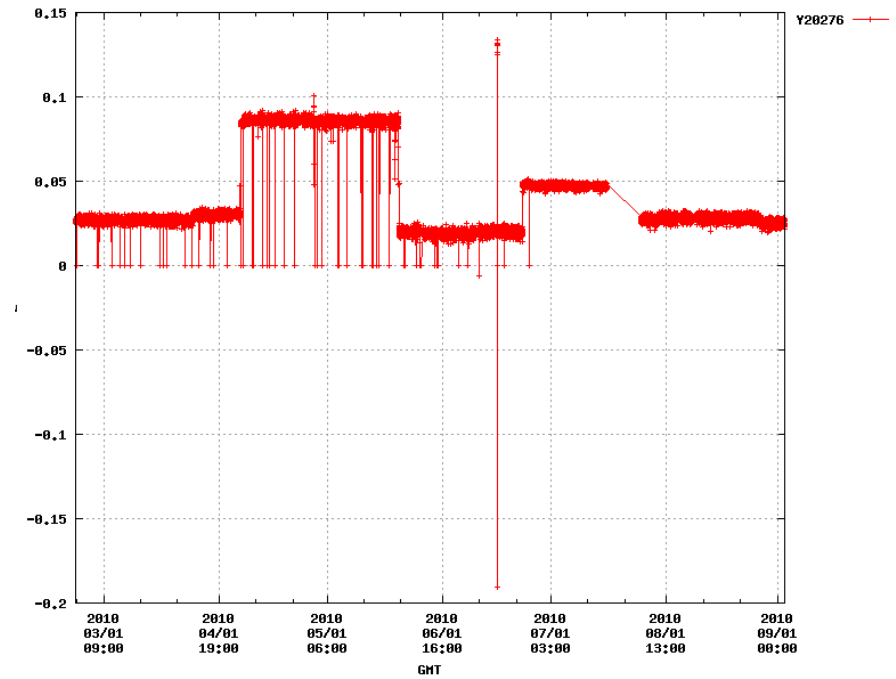
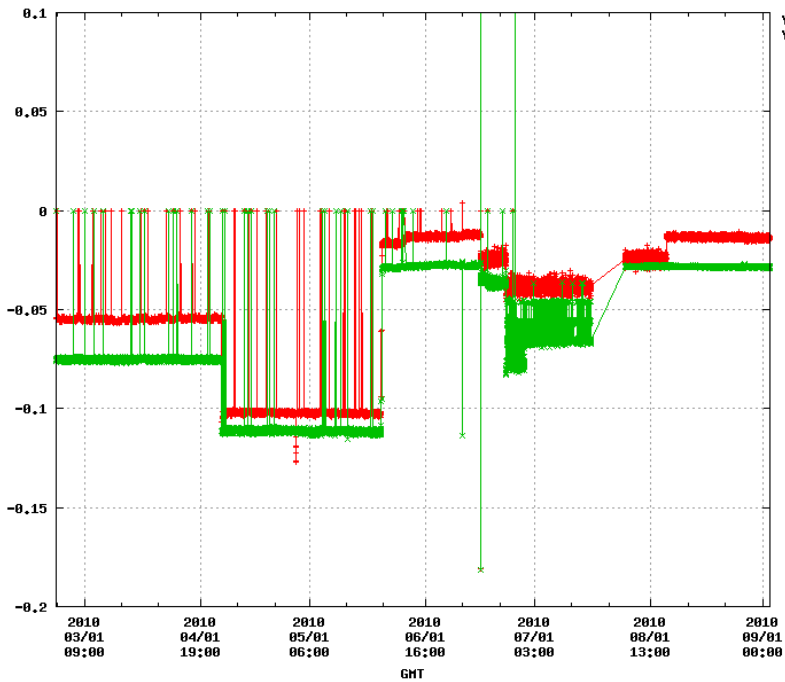
-1500.mm, phi-66. Buffer



-1000.mm, phi-150. Buffer



MAGNETIC FIELD IN BUFFER TANK FROM 2/21~9/2



RECENT PROCESS

- Temperature Auto-warning system is done in lab
- Filling process (especial target tank) depend on the temperature changing in the tanks.
- Magnetic monitoring is proved working well in IV lid open period.
- IV and Buffer thermal data-> thermal model of detector-> proton number.



FINAL FITTING PROCESS

○ Constructing a Covariance Matrix

$$P = C \exp\left(-\frac{1}{2} \chi^2\right), \chi^2 = \sum_{ij} \left(\frac{m_i}{t_i} - 1\right) V_{ij}^{-1} \left(\frac{m_j}{t_j} - 1\right), V_{ij} = \delta_{ij} \sigma_i^2 + \sum_k \Delta_{ik} \Delta_{jk}$$

- Parameters determined directly from calib. Data
- Parameters should be simultaneously varied in MC to match calib. Data



CONCLUSION

- 3 mass eigenstate model is the simplest model to explain three flavor neutrino oscillation
- Reactor neutrino experiment advantages
 - Doesn't depend on the δ -CP phase
 - Uniform antineutrino flux
 - no CP if $\sin^2 \theta_{13} \rightarrow 0$
- Double Chooz experiment
 - Near detector \rightarrow raise sensitivity from Sensitivity models $\sin^2 \theta_{13} \cong 0.02 \sim 0.015$
 - Buffer range \rightarrow reduce background and threshold energy



CONCLUSION

○ Recent work

- Environmental monitoring
 - Hardware and database are done
 - Thermal information analysis for filling system
 - Warning system
 - Thermal model of detector-> number of protons in detector
- Energy scale and final fitting
 - Parameters determined directly from calib. Data
 - Constructing a Covariance Matrix



REFERENCE

- “Double Chooz: A Search for the Neutrino Mixing Angle θ_{13} ” F. Ardellier et al. http://arxiv.org/PS_cache/hep-ex/pdf/0606/0606025v4.pdf
- “Letter of Intent for Double-CHOOZ: a Search for the Mixing Angle θ_{13} ” F. Ardellier et al. http://arxiv.org/PS_cache/hep-ex/pdf/0405/0405032v1.pdf
- “Reactor-based Neutrino Oscillation Experiments” Carlo Bemporad et al. arXiv:hep-hp/0107277 v1
- “Systematic limit on $\sin^2 2\theta_{13}$ in neutrino oscillation experiments with multi-reactor” H. Sugiyama et al.
- “Error Estimates on Parton Density Distributions” M Botje et al.
- “A precise determination of the Kamland energy scale” Timothy M. Classen thesis.
- “Neutrino Phenomenology Facts, and Question” Boris Kayser 2009 neutrino summer school



SYSTEMATIC ERRORS

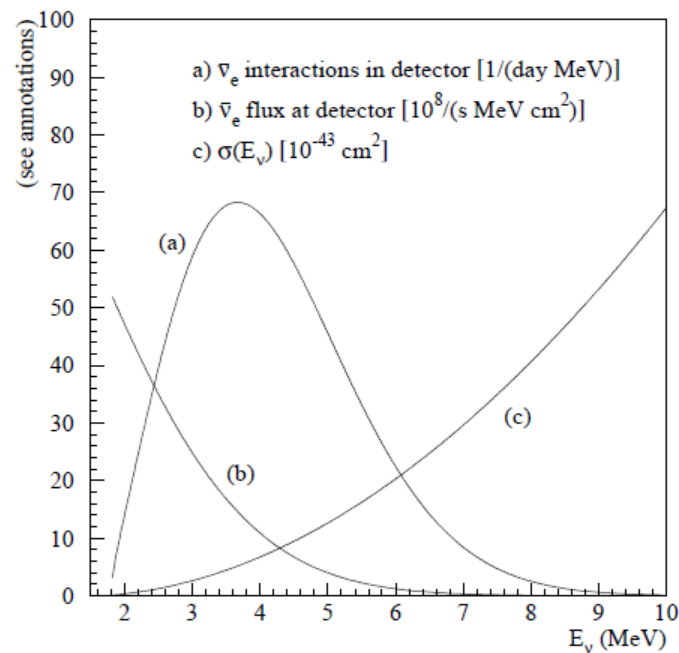
- Global normalization error: uncertainty of antineutrino flux and detector cross section(no impact)
- Relative normalization error: experiment error, uncertainties on the detector design and event select cuts.
- Spectral shape error: antineutrino spectrum shape ->the energy bin we take.
- Energy scale: energy scale calibration uncertainty of visible energy(seen in the detector)
- Background subtraction step: only one uncorrelated error

Error type	
Global normalization	2%
Relative normalization	0.6%
Spectrum shape	2%
Energy scale	0.5%
background	0.5%

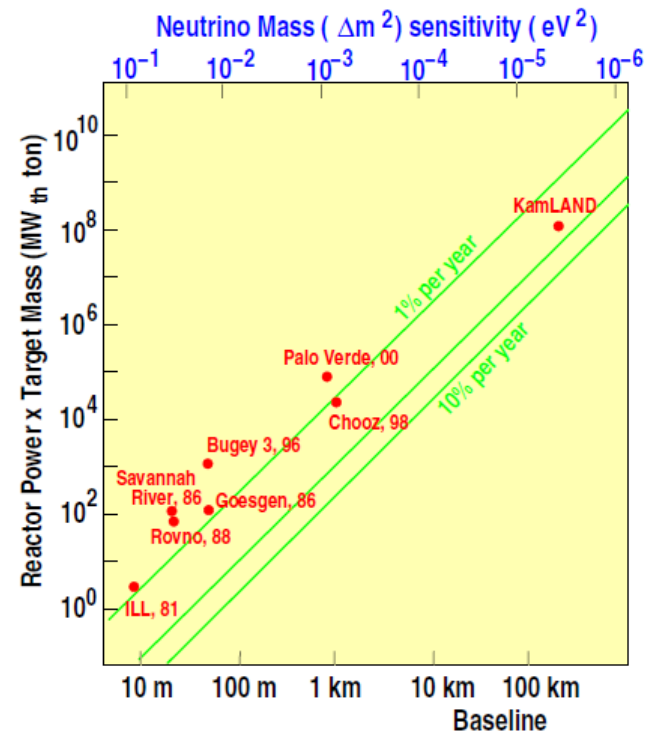


REACTOR NEUTRINO EXPERIMENT PROPERTIES

flux and cross section



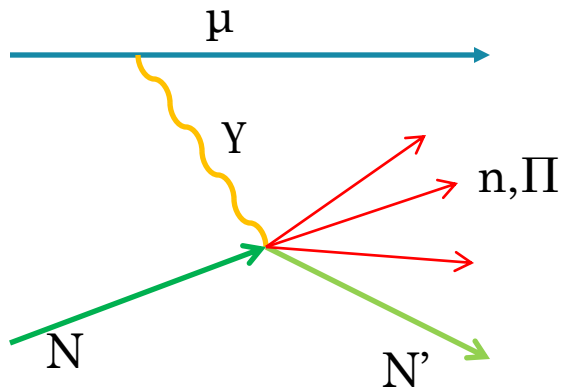
Neutrino Δm^2 sensitivity



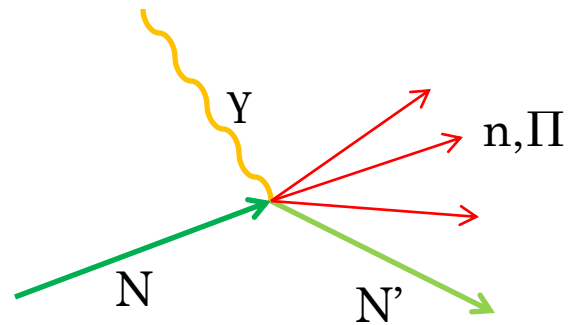
“Reactor-based Neutrino Oscillation Experiment” Carlo Bemporad et al.

MUON SPALLATION AND PHOTO-ABSORPTION

○ spallation



○ Photo-absorption



CROSS SECTION FOR NEUTRINO ABSORPTION

- Reaction rate

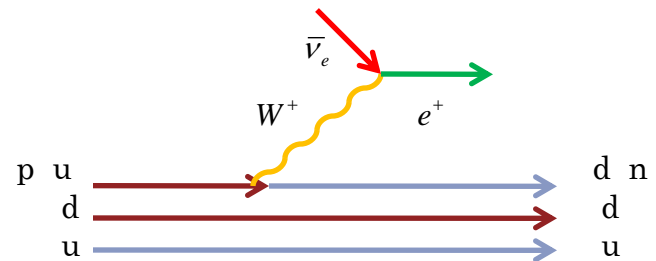
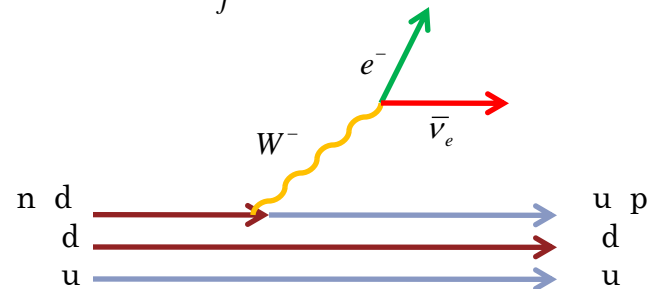
$$dW_r \equiv JN \frac{d\sigma_r(\theta, \phi)}{d\Omega} d\Omega = \frac{2\pi}{V} \rho(E_f) |m_{if}|^2, \rho(E_f) = \frac{Vq_f^2}{8\pi^3 v_f} d\Omega$$

- $n \rightarrow p + e^- + \bar{\nu}_e$ cross section

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar c} G_F^2 \langle |m_{fi}|^2 \rangle \left[\frac{p_e E_e}{(2\pi\hbar)^3 c^2} \right] \left[\frac{p_{\bar{\nu}}^2}{(2\pi\hbar)^3 c^2} \right]$$

- $\bar{\nu}_e + p \rightarrow n + e^+$ cross section

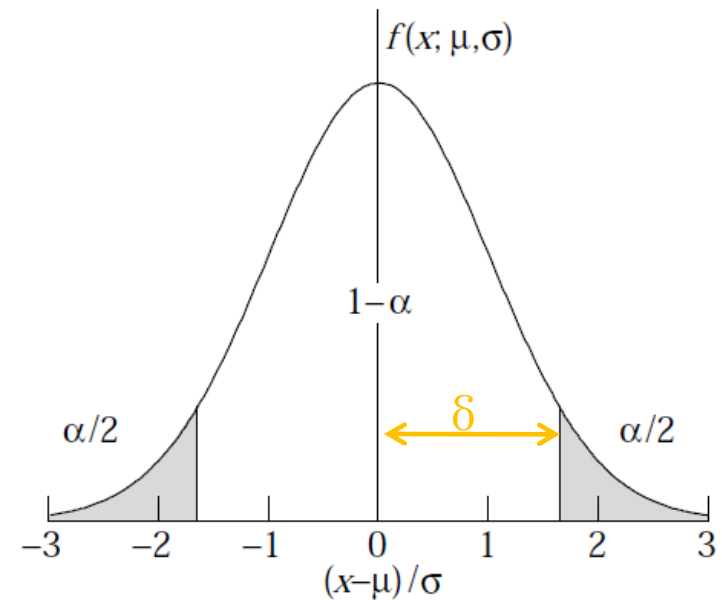
$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar c} G_F^2 \langle |m_{fi}|^2 \rangle \left[\frac{p_{e^+} E_{e^+}}{(2\pi\hbar)^3 c^2} \right]$$



CONFIDENCE INTERVAL

- 90%=1.28 σ
- 68.27%~ σ

α	δ
0.3173	1 σ
0.2	1.28 σ
0.1	1.64 σ
0.05	1.96 σ
0.01	2.58 σ
0.001	3.29 σ
0.0001	3.89 σ



From: Revised September 2009 by G. Cowan (RHUL).

THE METHOD OF LEAST SQUARES

- LS estimator -> minimum of

$$\chi^2(\theta) = (m - t(\theta))^T V^{-1} (m - t(\theta))$$

- χ^2 is equal to zero -> LS estimators

$$t(x_i; \vec{\theta}) = \sum_{j=1}^m \theta_j h_j(x_i); \hat{\theta} = (H^T V^{-1} H)^{-1} H^T V^{-1} \vec{m} \equiv D \vec{m} = U \vec{g}$$

$$U = D V D^T (H^T V^{-1} H)^{-1}; g_i = \sum_{j,k=1}^N m_j h_i(x_k) (V^{-1})_{jk}$$



MONTE CARLO TECHNIQUES

- Sample random variables governed by complicated probability density function
- $F(x)$ is a uniform distribution $(0,1)$
- $f(x)$ desired probability density function.

$$u = F(x)$$

$$x = F^{-1}(u)$$

