Homework 4

Due in class Feb. 10

From T&M: 1-31

2. Let's revisit problem 2-39, but this time with a physically reasonable force. Let's use

$$F = -\alpha \sinh \beta v.$$

This force has the same large v behavior as that given in 2-39, but actually goes to zero for $v = 0$. The boat is launched with some initial velocity v_0.

(a) Find $v(t)$ and $x(t)$ (if you can't do the integrals analytically, consider looking in a book of integrals or numerically integrating the equation of motion). Plot both and compare with the results from problem 2-39. Discuss the differences physically (you'll probably want to plot $v(t)$ and $x(t)$ — along with those from 2-39 — past the point where the boat stops).

(b) The boat has an engine and provides its own constant thrust F_T. Find $v(t)$ and $x(t)$ for this case for an initial velocity of v_0.

(c) Find the terminal velocity v_t if one exists.

(d) Plot $v(t)$ and $x(t)$ and discuss the plots physically — consider the cases $v_0 < v_t$ and $v_0 > v_t$.

3. A particle of mass $m = 1$ kg undergoes simple harmonic motion in two dimensions. Assuming the angular frequencies in each dimension are equal, $\omega_x = \omega_y = 2\pi$ rad/s,

(a) Plot the trajectory for at least one full period for initial conditions $x_0 = 1$ m, $\dot{x}_0 = 0$ m/s, $y_0 = 0$ m, and $\dot{y}_0 = 1$ m/s.

(b) Repeat (a) for $x_0 = 0$ m, $\dot{x}_0 = -1$ m/s, $y_0 = 0$ m, and $\dot{y}_0 = 1$ m/s.

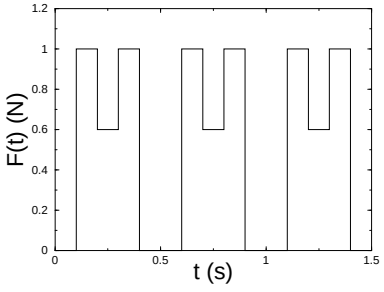
(c) Repeat (a) for $x_0 = 0.25$ m, $\dot{x}_0 = 0.5$ m/s, $y_0 = 0$ m, and $\dot{y}_0 = 1$ m/s.

4. Repeat problem 3, this time for

(a) $3\omega_x = 5\omega_y = 2\pi$ rad/s, and

(b) $\pi\omega_x = 5\omega_y = 2\pi$ rad/s.

5. Consider the periodic force in the figure below.



(a) Calculate the Fourier series corresponding to this force.

(b) Plot partial sums of your series from (a) including the first term, the first three terms, and the first five terms.

(c) Comment on the plots from (b) — how do they compare to the original force?