Homework 2

Due in class Sept. 4

From Griffiths: Probs. A.25 A.26, A.27, A.28

1. Show that the determinant of a matrix is unaffected by a unitary change of basis.

2. By considering the commutator, show that the following Hermitian matrices may be simultaneously diagonalized. Find the eigenvectors common to both and verify that under a unitary transformation to this basis, both matrices are diagonalized.

$$\Omega = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad \Lambda = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

Since Ω is degenerate and Λ is not, you must be prudent in deciding which matrix dictates the choice of basis.

3. A particle of mass m and charge q is placed in a static electric field of strength $\mathcal{E}$. The corresponding time-independent Schrödinger equation in configuration space is

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - q\mathcal{E}x\right]\psi(x) = E\psi(x).$$

(a) Is the spectrum of eigenvalues continuous or discrete? Explain your answer.

(b) Find the physically acceptable eigenfunctions $\psi(x)$. HINT: You will need to use some special functions!

(c) Can you normalize your solutions from (b)? Explain your answer.

(d) By transforming the Schrödinger equation to momentum space, show that the momentum space eigenfunctions $\psi(p)$ can be obtained by elementary methods.

(e) **Extra credit:** Use the Fourier transform to transform your solution from (d) to those from (b).

4. Given the following Hamiltonians in 3D space, state (i) what physical system the Hamiltonian might describe and (ii) the conserved quantities (if any):

$$H = -\frac{\hbar^2}{2m}\nabla^2 \tag{1}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + eEz \tag{2}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} \tag{3}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2(x^2+y^2+z^2) \tag{4}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \alpha\sin(\omega t)r^2 \tag{5}$$

where m, e, E, Z, ω, and α are constants.

Extra credit: What quantitities are conserved for the following Hamiltonian

$$H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + \frac{1}{2}\mu\omega^2|\mathbf{r}_1 - \mathbf{r}_2|^2$$

with m_i and ω constants?