Homework 11

Due in class Nov. 27

From Griffiths: 7.1 and 7.14

Extra Credit: 7.20 from Griffiths

1. Consider the spin-orbit interaction in the hydrogen atom,

$$W = \frac{e^2}{2m^2c^2}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}$$

where e and m are the charge and mass of the electron, $\mathbf{L}$ is its orbital angular momentum, $\mathbf{S}$ is its spin, and r is its distance from the proton.

(a) Calculate the first-order energy shift of the ground state.

(b) Calculate the first-order energy shifts of the $n = 2$ states.

This correction is one of the relativistic corrections to the simple hydrogen energy spectrum that you're used to and contributes to the fine structure of hydrogen. (HINT: Notice that $J^2 = L^2 + S^2 + 2\mathbf{L}\cdot\mathbf{S}$.)

2. Consider two *different* spin-$\frac{1}{2}$ particles whose Hamiltonian is completely specified as $H = c\mathbf{S}_1 \cdot \mathbf{S}_2$ where c is a real constant. Now, these spins are subject to

$$H_1 = \mathcal{F}_0(S_{1y} + S_{2y})\cos\omega_0 t \qquad -\frac{T}{2} \le t \le \frac{T}{2}$$

is applied. The pulse is long, $T = 20\frac{2\pi}{\omega_0}$, and $\omega_0 = 2c\hbar$.

(a) Assuming the initial state is the ground state, calculate the probabilities to be in each of the excited states after the pulse in lowest order perturbation theory. Can you identify a selection rule for this perturbation?

(b) Calculate the probability to remain in the ground state to the same order in the perturbation. Is unitarity preserved?

(c) You should be able to obtain numerical answers to (a) and (b) (with $\mathcal{F}_0$). For what values of $\mathcal{F}_0$ are your answers valid?

(d) Repeat (a)–(c) if the perturbation is instead

$$H_1 = \mathcal{F}_0(S_{1z} + S_{2z})\cos\omega_0 t \qquad -\frac{T}{2} \le t \le \frac{T}{2}.$$

3. The motion of two *identical* spin-$\frac{1}{2}$ particles is governed by the Hamiltonian

$$H = c\mathbf{S}_1 \cdot \mathbf{S}_2.$$

(a) Find the eigenenergies and eigenstates. You may ignore the spatial degrees of freedom.

(b) The perturbation

$$H' = \gamma(S_{1x} + S_{2x})$$

is applied to the system. Calculate the energy shifts in lowest order perturbation theory.

(c) The perturbation

$$H' = \gamma(S_{1z} + S_{2z})$$

is applied to the system. Calculate the energy shifts in lowest order perturbation theory. Compare your answers to (b) and comment on their relation.