Homework 1

Due beginning of class Aug. 28

From Griffiths: Probs. 3.22, 3.23, 3.24, A.2, A.8, A.9, A.13, A.14, A.15

1. Consider three elements from the vector space of real 2×2 matrices:

$$|1\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad |3\rangle = \begin{pmatrix} -2 & -1 \\ 0 & -2 \end{pmatrix}$$

   (a) Are they linearly independent? Support your answer with details. (Note we are calling these matrices vectors and using kets to represent them to emphasize their role as elements of a vector space.)

   (b) Does the set $\{|1\rangle, |2\rangle, |3\rangle\}$ span the space of real 2×2 matrices? In other words, does it constitute a basis for this space?

3. An operator Ω is given by the matrix:

$$\Omega = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

What is its action?

4. Given that Ω and Λ are Hermitian, what can you say about the following combinations:

   (a) $\Omega\Lambda$

   (b) $\Omega\Lambda + \Lambda\Omega$

   (c) $[\Omega, \Lambda]$

   (d) $i[\Omega, \Lambda]$.

Are they Hermitian, anti-Hermitian, or neither?

6. Verify that the following matrices are unitary:

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad B = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}.$$

Verify also that the determinant is of the form $e^{i\theta}$ in each case. Are any of the above matrices Hermitian?

Remember:

- Write your solutions clearly and logically, showing your work.
- Write your solutions on your own. If you worked with someone, list their names.
- Cite any sources that you used (reference book, web site, another instructor, Mathematica, etc.)