Homework 9

Due in class Oct. 28

From Shankar: Exercises 17.3.2, 18.2.1, 18.2.4.

Note that the spin matrices needed for 17.3.2 are

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = i\frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Don't worry about the spin aspect of this problem, you need only construct the 3×3 Hamiltonian matrices H_0 and H_1. Also, the language "stable under the perturbation" just means to do degenerate perturbation theory if you need to.

4. Given the Hamiltonian

$$H = \begin{pmatrix} E_1 & \gamma \\ \gamma & E_2 \end{pmatrix}$$

with γ real,

(a) Find the exact eigenvalues of H.

(b) Find the approximate eigenvalues using perturbation theory to the lowest nonvanishing order.

(c) Expand your answers from (a) in a Taylor series in an appropriate small parameter and compare with your results from (b). Do your results make sense?

5. Consider the infinite square well:

$$V(x) = \begin{cases} 0 & \text{if } |x| \le \frac{a}{2} \\ +\infty & \text{otherwise} \end{cases}.$$

(a) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form

$$H_1 = \varepsilon \frac{2|x|}{a}.$$

State the conditions under which this result is valid.

(b) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form

$$H_1 = \varepsilon \frac{2x}{a}.$$

State the conditions under which this result is valid.

(c) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form

$$H_1 = \begin{cases} 0 & \text{if } x \le 0 \\ \varepsilon \frac{2x}{a} & \text{otherwise} \end{cases}.$$

State the conditions under which this result is valid.

(d) Are your results from (a), (b), and (c) consistent? Do they make physical sense (in comparison with one another)? In particular, discuss the role of symmetry.

6. An electron is moving in a 3D simple harmonic oscillator of frequency ω. A constant magnetic field of magnitude $\mathcal{B}_0$ is applied along the z-axis, and we will consider the effects of the diamagnetic term:

$$H' = \frac{e^2 \mathcal{B}_0^2}{8 m_e}(x^2 + y^2).$$

(a) Calculate the energy shift of the ground state in first order perturbation theory.

(b) Calculate the energy shifts for the first excited state ($E = \frac{5}{2}\hbar\omega$) in first order perturbation theory.

(c) Calculate the energy shifts for the second excited state ($E = \frac{7}{2}\hbar\omega$) in first order perturbation theory.

(d) For what values of $\mathcal{B}_0$ are your results in (a)–(c) valid? If the experimentalist doesn't want to worry about this effect, will they have to shield their quantum dots from, say, the earth's magnetic field?

(e) Interpret your results in (a)–(c) physically. Do the directions of the energy shifts make sense? Are they consistent with the corresponding wave functions?