Homework 5

Due in class Sept. 23

From Shankar: Exercise 5.2.2, and

2. Consider the potential

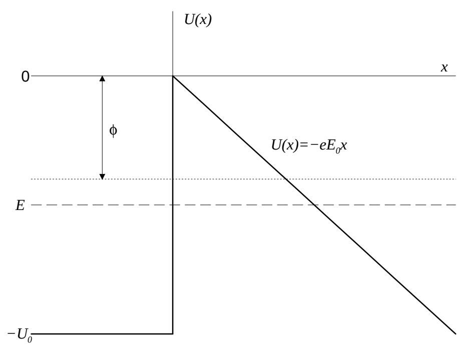
$$V(x) = V_0 a \delta(x).$$

(a) Calculate the reflection and transmission coefficients R and T.

(b) Do R and T depend on whether the potential is attractive or repulsive? If so, how?

(c) Plot or sketch R and T. Do they make sense physically? Explain.

3. Field emission occurs when an electron is pulled from a metal surface by a strong electric field E_0. This process can be modeled by the potential $U(x)$ in the figure. The electron is bound inside the metal, $x < 0$, with energy E. The work function of the metal ϕ is the energy required to extract an electron at the Fermi energy E_F, *i.e.* the least amount of energy that will free an electron in the absence of an electric field. Electrons with lower (more negative) energies, however, can also tunnel out.



(a) Based on qualitative arguments, what total energy do you expect most electrons will have upon leaving the metal?

(b) Quantitatively estimate the tunneling probability as a function of energy.

(c) Plot your result from (b) and briefly discuss it physically.

4. Consider a free electron whose initial wave function is given by the Gaussian wave packet of Eq. (5.1.14) on p. 154. (See also HW 3 #5 and HW 4 #5.)

(a) Use units such that $\hbar = m = 1$, an initial width of $\Delta = 1$, and an initial velocity of 0.1, find a way to plot or sketch the probability density $|\psi(x,t)|^2$ at several (≈ 10) times t from -10 to 10.

(b) Describe the main features of the wave packet's evolution, especially the comparison with your expectations from the classical motion of a free particle and from the uncertainty principle.

(c) Plot $\langle X\rangle(t)$ and $\Delta X(t)$ under the conditions of (a). Explain whether (and why) these are consistent with the plots in (a).

(d) For the conditions in (a), plot the probability density $|\tilde{\psi}(p,t)|^2$ and discuss it's features (*i.e.* $\langle P\rangle(t)$ and $\Delta P(t)$ and the connection with classical mechanics and the uncertainty principle).

(e) Calculate the mean energy of the wave packet $E(t)$ and the width of the wave packet in energy space $\Delta E(t)$.

(f) Now, imagine adding a potential step and scattering your Gaussian wave packet from it. Discuss what you think will happen and try to support your expectations as quantitatively as possible. You should consider mean energies both above and below the step. HINT: Think about R and T for this problem.

5. Consider the potential

$$V(\phi) = V_0 a \delta(x-a) + \begin{cases} V_1 & \text{if } x \le 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) This potential differs qualitatively from the other potentials in this assignment. Sketch the potential and discuss what new features in R and T might arise for it. Make quantitative estimates for the energy positions of these features.

(b) Calculate R and T for all possible energies E. Plot your results for both left and right incidence. Were your expectations in (a) correct?

(c) Do your expressions for R and T make sense? Check the limits in energy and check the limits $V_0 \to 0$ or $V_1 \to 0$.

Extra Credit

A particle of mass m is confined to move around a ring of radius a. There is thus only one degree of freedom — the angle ϕ. The particle sees a potential given by

$$V(\phi) = \begin{cases} +V_0 & \text{if } 0 \le \phi \le \frac{\pi}{2} \\ 0 & \text{otherwise.} \end{cases}$$

What are the allowed energies for this system?