Homework 3

Due in class Sept. 9

From Shankar: Exercises 4.2.1, 4.2.2, 4.2.3, and

4. Consider a particle of mass m in the potential:

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ +\infty & \text{otherwise.} \end{cases}$$

$|\phi_n\rangle$ are the eigenstates of the Hamiltonian H of the system, and their eigenvalues are

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \ldots$$

The state of the particle at the instant $t = 0$ is:

$$|\psi(0)\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle + a_3|\phi_3\rangle.$$

(a) What is the probability, when the energy of the particle in the state $|\psi(0)\rangle$ is measured, of finding a value smaller than $\frac{2\pi^2\hbar^2}{ma^2}$?

(b) What is the mean value and what is the root-mean-square deviation of the energy of the particle in the state $|\psi(0)\rangle$?

(c) Calculate the state vector $|\psi(t)\rangle$ at the instant t. Do the results found in (a) and (b) above at the instant $t = 0$ remain valid at an arbitrary time t?

(d) When the energy is measured, the result $\frac{\pi^2\hbar^2}{2ma^2}$ is found. After the measurement, what is the state of the system? What is the result if the energy is measured again?

5. Consider a free particle whose initial wave function is given by the Gaussian wave packet

$$\psi(x, 0) = \mathcal{N} e^{ik_0 x} e^{-\frac{1}{2}(\frac{x}{\Delta})^2}$$

(a) Find the normalization constant $\mathcal{N}$.

(b) Calculate the corresponding momentum space wave function and write the coordinate space wave function as an expansion on momentum space basis functions.

(c) Using your expansion from (b), write down the wave function at an arbitrary time t. Then, carry out the Fourier transform (do the momentum space integral) and find the explicit form of $\psi(x, t)$.

(d) Use your wave function from (c) to calculate $\langle X\rangle(t)$ and $\Delta X(t)$ (you can compare with the answers in the text).

Supplemental reading:

Skim the Examples in Chap. 4 to make sure that you understand them.