Homework 2

Due in class Sept. 2

From Shankar: Exercises 1.7.2, 1.8.3, 1.8.7, 1.8.10, and

5. Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}$$

(a) Is it Hermitian?

(b) Find its eigenvalues and eigenvectors.

(c) Verify that $U^\dagger AU$ is diagonal, U being the matrix of eigenvectors of A.

6. Given the following Hamiltonians in 3D space, state the conserved quantities (if any):

$$H = -\frac{\hbar^2}{2m}\nabla^2 \tag{1}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + eEz \tag{2}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r} \tag{3}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2(x^2+y^2+z^2) \tag{4}$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \alpha\sin(\omega t)r^2 \tag{5}$$

where m, e, E, Z, ω, and α are constants.

Extra credit: What quantitities are conserved for the following Hamiltonian

$$H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + \frac{1}{2}m\omega^2|\mathbf{r}_1 - \mathbf{r}_2|^2$$

with m_i and ω constants?

Supplemental reading:

Review Chap. 2, especially Hamiltonians and Sec. 2.8.
Review Chap. 3 — motivation for quantum mechanics.