Homework 11

Due in class Nov. 11

From Shankar: Exercises 10.3.2, 10.3.3, 10.3.4, and

5. Consider two identical particles of mass m in an infinite square well of width a (see exercise 10.3.4).

(a) Assuming the particles are *bosons*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

(b) Assuming the particles are *fermions*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

(c) Using Mathematica or any similar program, plot the wave functions you obtained in (a) and (b). These are functions of two variables, so you will need to plot contours or a surface. Comment on any general features you find.

6. Consider two identical particles of mass m in an infinite square well of width a (see exercises 7 and 10.3.4). Now, add the interparticle interaction

$$H_1 = \frac{\lambda}{a^2}(x_1 - x_2)^2$$

as a perturbation.

(a) Assuming the particles are *bosons*, calculate the first order energy shift of the two lowest states.

(b) Assuming the particles are *fermions*, calculate the first order energy shift of the two lowest states.

(c) Where possible, compare the energy shifts from (a) and (b) and discuss the physical basis of their difference. That is, why do bosons shift more/less than fermions? You might want to refer back to your pictures in 7(c).

7. We have said many times that an arbitrary function can be expanded on a complete basis set. Let's see just how it works:

(a) The set of monomials $\{1, x, x^2, x^3, \ldots\}$ can be thought of as a basis set (even though it's not orthonormal). Thinking this way, when you do a Taylor series, you're just expanding on a basis set. We all know the Taylor series expansion of $\cos x$. Plot $\cos x$ along with the first four partial sums of the Taylor series, *i.e.* keeping just the first term, the first two terms, the first three, etc. For what values of x would you consider each approximation to $\cos x$ valid?

It's a little easier to develop intuition for expansions on orthonormal basis sets, so let's do an example. In the last homework, you treated the sudden expansion of an infinite square well from $0 \le x \le a$ to $0 \le x \le 2a$. You solved the problem by writing

$$\psi(x,t) = \sum_{n=1}^{\infty} a_n e^{-i\frac{E_n}{\hbar}t} \phi_n^{(2a)}(x)$$

and finding the expansion coefficients a_n from the initial condition $\psi(x, t=0) = \phi_1^{(a)}(x)$.

(b) Calculate the first 10 expansion coefficients. You probably want to write the integral in a general form, then substitute the necessary values afterwards, if possible. Plot the square of the expansion coefficients (why the square? what is the physical interpretation of the square?). Comment on any pattern you notice. Can you deduce what happens to the coefficients in the limit of large n? Is the $n \to \infty$ behavior sensible?

(c) Plot $\psi^N(x, t=0)$ for $N = 1, 2, 3, 4$ where

$$\psi^N(x,t) = \sum_{n=1}^{N} a_n e^{-i\frac{E_n}{\hbar}t} \phi_n^{(2a)}(x).$$

When $N \to \infty$, you should obtain the initial state $\phi^{(a)}(x)$, right? Comment on the differences between $\psi^N(x, t=0)$ and initial state. Also, comment on any trends you see.

(d) Estimate how large N should be to recover 99% of the initial state. That is, for what N is the following satisfied:

$$\sum_{i=1}^{N} |a_i|^2 = 0.99.$$

(e) For $N = 3$, plot $|\psi^N(x,t)|^2$ for several values of t from 0 to $t > \frac{2ma^2}{\pi\hbar}$. Comment on the behavior of the system. Is it what you had in mind when you did the problem on the last homework? Does it make sense physically? Why?