Homework 9

Due in class Nov. 6

From Shankar: Exercises 10.3.2, 10.3.3, 10.3.4, 12.5.3, and

5. Consider two identical particles of mass m in an infinite square well of width a (see exercise 10.3.4).

   (a) Assuming the particles are *bosons*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

   (b) Assuming the particles are *fermions*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

   (c) Using Mathematica or any similar program, plot the wave functions you obtained in (a) and (b). These are functions of two variables, so you will need to plot contours or a surface. Comment on any general features you find.

6. Consider two identical particles of mass m in an infinite square well of width a (see exercises 7 and 10.3.4). Now, add the interparticle interaction

$$H_1 = \frac{\lambda}{a^2}(x_1 - x_2)^2$$

as a perturbation.

   (a) Assuming the particles are *bosons*, calculate the first order energy shift of the two lowest states.

   (b) Assuming the particles are *fermions*, calculate the first order energy shift of the two lowest states.

   (c) Where possible, compare the energy shifts from (a) and (b) and discuss the physical basis of their difference. That is, why do bosons shift more/less than fermions? You might want to refer back to your pictures in 7(c).

7. Consider a particle in a state described by

$$\psi(x) = \mathcal{N}\left(x + ye^{i\frac{\pi}{4}} - z\right)e^{-\alpha r}$$

where $\mathcal{N}$ is a normalization constant and α is a positive real constant. Following the general method outlined in Exercise 12.5.13, calculate:

   (a) The probability that a measurement will yield $\ell = 0, 1, 2, \ldots$.

   (b) The probability that a measurement will yield $m = \ldots, -1, 0, 1, \ldots$.

   (c) The probability that a measurement will yield $\ell = 1$ and $m = -1$.

8. Since L^2 and L_z commute with Π (parity), they should share a basis with it. Verify that

$$\Pi Y_{\ell m} = (-)^{\ell} Y_{\ell m}.$$

(Hint: first show that $\theta \to \pi - \theta$ and $\phi \to \phi + \pi$ under parity. Then use the properties of spherical harmonics.)