Homework 8

Due in class Oct. 23

1. Given the Hamiltonian
$$H = \begin{pmatrix} E_1 & \gamma \\ \gamma & E_2 \end{pmatrix}$$
   (a) Find the exact eigenvalues of H.

   (b) Find the approximate eigenvalues using perturbation theory (at least second order!).

   (c) Expand your answers from (a) in a Taylor series in an appropriate small parameter and compare with your results from (b). Do your results make sense?

2. Consider an infinite square well
$$V(x) = \begin{cases} 0 & \text{if } |x| \leq \frac{a}{2} \\ +\infty & \text{otherwise} \end{cases}$$
subject to the time-dependent perturbation
$$H_1(t) = \lambda X \sin \omega t \qquad 0 \leq t \leq T.$$
   (a) Calculate and plot (as a function of ω) the transition probabilities to the first, second, and third excited states assuming the system in the ground state initially. Set
$$\omega = \frac{8\pi^2\hbar^2}{2ma^2}$$
   and plot the same probabilities as a function of T. Explain the results in all cases.

   (b) For the cases in (a), what is the probability (to the same order as in (a)) that the ground state survives the perturbation?

   (c) Assume now that the initial state of the system is an equal admixture of the ground and first excited state. Repeat part (a), making sure to explain the results and contrast them to those in (a).

3. Consider again the infinite square well:
$$V(x) = \begin{cases} 0 & \text{if } |x| \leq \frac{a}{2} \\ +\infty & \text{otherwise} \end{cases}.$$
   (a) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form
$$H_1 = \varepsilon \frac{2|x|}{a}.$$
   State the conditions under which this result is valid.

   (b) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form
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   (c) Calculate the energy shifts to first order for the three lowest states for a perturbation of the form
$$H_1 = \begin{cases} 0 & \text{if } x \leq 0 \\ \varepsilon \frac{2x}{a} & \text{otherwise} \end{cases}.$$
   State the conditions under which this result is valid.

   (d) Are your results from (a), (b), and (c) consistent? Do they make physical sense (in comparison with one another)?