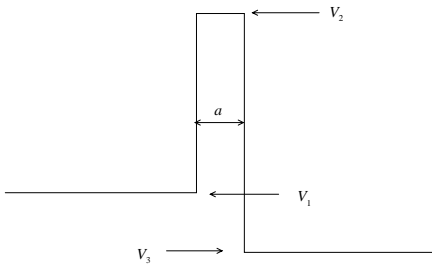


Homework 5

Due in class Sept. 25

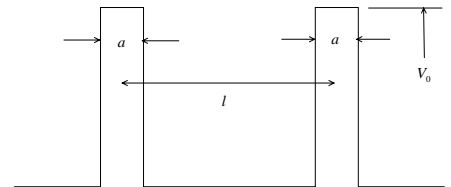


1. Consider a particle of mass m subjected to the potential in the figure to the left. You may assume that $V_2 - V_1 = 3\Delta V$ where $\Delta V = V_1 - V_3$.

- (a) Calculate the reflection and transmission coefficients as a function of energy for all energies (you should consider both incidence from the left and from the right).
- (b) Calculate the tunneling probability approximately as a function of energy.
- (c) Plot your results from (a) and (b), comparing the exact and approximate results. Comment on whether the agreement (or disagreement) is as you expected.
- (d) Discuss physically any structure in R and T that you might have found. Give evidence for your statements.

2. Given the potential shown at the right:

- (a) Calculate the reflection R and transmission T coefficients for a particle incident from the left for all energies.
- (b) Calculate the phase shift ϕ for transmission, also for all energies.
- (c) Plot T and ϕ and relate any features in T to features in ϕ and to the wave function. Comment on their physical interpretation.



3. Consider the following potential:

$$V(x) = V_0 \left[\left(\frac{x}{\alpha} \right)^2 - 1 \right]^2$$

where α and V_0 are constants. A potential of this form is a good model for the position of the nitrogen atom in ammonia, NH_3 . The shape of ammonia can be pictured as a pyramid with a triangular base — the H's lie at the corners of the base, and the N at the top. There is no reason, however, for the nitrogen to stay above the H's, and it can tunnel through the base to be on bottom.

- (a) Sketch or plot the potential and your guess for the ground and first excited state wave functions.
- (b) For a particle localized on one side of the barrier, the potential appears nearly harmonic near the minimum. Find the effective harmonic potential for such a particle.
- (c) Construct an approximate ground and first excited state wave function as a linear combination of the ground states of the right- and left-side simple harmonic oscillator (SHO) potentials from (b). Calculate the energies of your approximate states from the expectation value of the Hamiltonian with the full potential. Use $V_0 = \frac{6\hbar^2}{m\alpha^2}$ and $V_0 = \frac{8\hbar^2}{m\alpha^2}$. Explain the change in energies between these two values of V_0 . The energies of the SHO states were the same — why are the energies of your approximate states different?
- (d) Estimate the time it takes for a particle localized to the left of the barrier to appear on the right side (use the values of V_0 from (c)). (HINT: Your approximate eigenstates will evolve in time with a phase approximately given by their energies).

4. Consider a pair of square wells of width a and depth $-V_0$ that are separated by a distance l (center to center), i.e. change the barriers in the figure from Prob. 2 to wells. Such a potential is qualitatively very much like what an electron might see in a diatomic molecule (say H_2^+). The electron moves much faster than the heavy protons, so they appear essentially fixed in space.

- (a) Calculate the bound state wave functions and energies as a function of l . You may want to use the program available on the class web site, but you will need to put in the potential yourself. Plot the energies as a function of l — these turn out to be effective potential curves for the motion of the protons (once the proton-proton repulsion is added). Explain the behavior of the curves as a function of l , especially the limits $l \rightarrow 0$ and $l \rightarrow \infty$. You may want to plot the wave function at a couple of representative values.
- (b) Add one more potential well a distance l from its neighbor. Repeat part (a).
- (c) Repeat for $N=4,5, \dots$ potential wells to deduce what happens to the energy levels for $N \rightarrow \infty$ and explain your conclusion.

5. For the potential

$$V(x) = -V_0 a (\delta(x - a) + \delta(x + a))$$

with a and V_0 positive, real constants,

- (a) Calculate the bound state wave functions and energies and plot the wave functions. Comment on their symmetry in relation to their energy.
- (b) Calculate the reflection and transmission coefficients for scattering from this potential. Plot T and discuss it — in particular, any similarities to Prob. 2.

Extra Credit

1. Continuing the computer problem from HW #4 ... For one of the above potentials (or one of your own choosing), numerically solve the time-dependent Schrödinger equation using the initial condition from HW #4.

- (a) Plot the probability density for a few representative times.
- (b) Describe the main features of the wave packet's evolution.
- (c) Explain why the behavior of the wave packet is reasonable in terms of what you've learned about one-dimensional problems (i.e. relate it to R and T , etc.).