

## Homework 3

Due in class Sept. 9

From Shankar: Exercises 4.2.1, 4.2.2, 4.2.3, and

4. Consider a particle of mass  $m$  submitted to the potential:

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ +\infty & \text{otherwise.} \end{cases}$$

$|\phi_n\rangle$  are the eigenstates of the Hamiltonian  $H$  of the system, and their eigenvalues are

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}, \quad n = 1, 2, 3, \dots$$

The state of the particle at the instant  $t = 0$  is:

$$|\psi(0)\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle + a_3|\phi_3\rangle + a_4|\phi_4\rangle.$$

- a. What is the probability, when the energy of the particle in the state  $|\psi(0)\rangle$  is measured, of finding a value smaller than  $\frac{3\pi^2 \hbar^2}{ma^2}$ ?
  - b. What is the mean value and what is the root-mean-square deviation of the energy of the particle in the state  $|\psi(0)\rangle$ ?
  - c. Calculate the state vector  $|\psi(t)\rangle$  at the instant  $t$ . Do the results found in a. and b. above at the instant  $t = 0$  remain valid at an arbitrary time  $t$ ?
  - d. When the energy is measured, the result  $\frac{8\pi^2 \hbar^2}{ma^2}$  is found. After the measurement, what is the state of the system? What is the result if the energy is measured again?
5. Consider a free particle whose initial wave function is given by the Gaussian wave packet of Eq. (5.1.14) on p. 154.
- a. Calculate the corresponding momentum space wave function and write the coordinate space wave function as an expansion on momentum space basis functions.
  - b. Verify explicitly the time-dependent wave function in Eq. (5.1.15).
  - c. Verify explicitly the expressions in the text for  $\langle X \rangle(t)$  and  $\Delta X(t)$ .

### Supplemental reading:

Skim the Examples in Chap. 4 to make sure that you understand them.