Homework 13

Due in class Dec. 11

1. Consider a particle of mass m moving in three dimensions according to the potential:

$$V(x) = \begin{cases} -V_0 & \text{if } 0 \leq r \leq a \\ 0 & \text{otherwise.} \end{cases}$$

   a. Solve the Schrödinger equation for positive energies E.

   b. Calculate and plot (as a function of energy) the phase shifts for the lowest three partial waves. Discuss whether your results make sense in the $E \to 0$ and $E \to \infty$ limits. Discuss any other features you find. For concreteness, use

$$\beta = \sqrt{\frac{2m}{\hbar^2} V_0} a = 8\pi,$$

   and plot your results to E/V_0=6.

   c. Calculate and plot the partial cross sections corresponding to the phase shifts you calculated in (b). Identify any features you find.

   d. Calculate and plot the total cross section that follows from (b) and (c). Have you included enough partial waves to obtain the total cross section converged to two digits over the entire energy range considered? Over what energy range do you think your answer is accurate to two digits?

2. The effect of magnetic fields on atoms is typically weak, such that it can be considered a perturbation on the fine structure levels. The effects of magnetic fields on atoms or molecules are collectively called the Zeeman effect.

We will assume that the magnetic field is $\mathbf{B} = B_0\hat{z}$, and the perturbation is written as

$$H' = \frac{\mu_B}{\hbar}(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}$$

where μ_B is the Bohr mageton. Physically, this perturbation describes the interaction of a magnetic dipole with the field $(-\mu \cdot \mathbf{B})$.

   a. Why did we choose $\mathbf{B}$ to be along the z-axis? Physically, does it matter?

   b. Take the unperturbed Hamiltonian to be

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{4\pi\epsilon_0 r} + \xi(r)\mathbf{L} \cdot \mathbf{S}.$$

   The last term is the spin-orbit interaction discussed in class. Calculate the energy level shifts due to the presence of the magnetic field using perturbation theory.

   c. For what values of B_0 is the Zeeman effect truly perturbative?

   d. Plot the total energies of the $1s_{\frac{1}{2}}$, $2s_{\frac{1}{2}}$, $2p_{\frac{1}{2}}$, and $2p_{\frac{3}{2}}$ levels as a function of B_0. You'll probably have to plot them on different scales. Identify in your plot where perturbation theory breaks down.