

Homework 10

Due in class Nov. 13

1. Given $\ell_1 = 1$ and $\ell_2 = 5$ (note that you may calculate any Clebsch-Gordan coefficients by whatever method you like):

- (a) Calculate the possible values of the total angular momentum $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$.
- (b) Write down the explicit expressions in terms of $|\ell_1 m_1\rangle |\ell_2 m_2\rangle$ for the states
 - (i) $|(15)3 - 3\rangle$
 - (ii) $|(15)4 - 3\rangle$
 - (iii) $|(15)6 - 6\rangle$
 - (iv) $|(15)65\rangle$.
- (c) Turn the expansion around and write the state $|11\rangle |5 - 4\rangle$ as an expansion on eigenstates of total angular momentum.

2. Consider a hydrogen atom in a constant external electric field $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$.

- (a) Calculate the correction to the ground state energy in first order perturbation theory.
- (b) Calculate the corrections to the $n = 2$ states in first order perturbation theory.
- (c) For the $m = 0$ states, argue that the direction of the energy shift relative to the unperturbed energies makes sense. (A good starting point is the electron density described by the wave functions.)
- (d) Sketch the energies you obtained in (a) and (b) as a function of the magnitude of the electric field \mathcal{E} . For what values of \mathcal{E} are these results valid?

The shifts you calculated are called the Stark effect; and the states, the Stark states. (HINTS: Write z in terms of Y_{lm} 's and notice that

$$\int d\Omega Y_{00}^* Y_{lm} Y_{l'm'} = \frac{1}{\sqrt{4\pi}} \int d\Omega Y_{lm} Y_{l'm'}$$

since $Y_{00} = 1/\sqrt{4\pi}$.)

3. Consider the spin-orbit interaction in the hydrogen atom,

$$W = \frac{e^2}{2m^2c^2} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3}$$

where e and m are the charge and mass of the electron, \mathbf{L} is its orbital angular momentum, \mathbf{S} is its spin, and r is its distance from the proton.

- (a) Calculate the first-order energy shift of the ground state.
- (b) Calculate the first-order energy shifts of the $n = 2$ states.

This correction is one of the relativistic corrections to the simple hydrogen energy spectrum that you're used to. (HINT: Notice that $J^2 = L^2 + S^2 + 2\mathbf{L} \cdot \mathbf{S}$.)

4. If we take the e - e interaction to be a perturbation in the helium atom, then the zero-th order eigenstates are just hydrogenic solutions with $Z = 2$ when spin-orbit and higher-order interactions are neglected.

- (a) Write down the ground singlet and triplet states (properly symmetrized and angular momentum eigenstates!). Argue that orbital and spin angular momenta are separately conserved.
- (b) Calculate to first order the shifts to the states in (a) due to the electron-electron interaction. Do the directions of the shifts make sense? (HINT: Consider the two-electron density.)
- (c) Repeat (a) and (b) for the H^- ion. Is H^- stable in this approximation, or is it more favorable to break up into $\text{H} + e$?

5. Using Fermi's Golden Rule, calculate the $1s \rightarrow 2\ell$ transition rates for hydrogen induced by linearly polarized light. In general, which transitions can be induced by linearly polarized light starting from the ground state?