Homework 9

Due in class Nov. 14

From Shankar: Exercises 10.1.1, 10.1.2, 10.1.3, 10.2.2, and

8. Given the Hamiltonian

$$H = \begin{pmatrix} E_1 & \gamma \\ \gamma & E_2 \end{pmatrix}$$

(a) Find the exact eigenvalues of H.

(b) Find the approximate eigenvalues using perturbation theory (at least second order!).

(c) Expand your answers from (a) in a Taylor series in some small parameter and compare with your results from (b). Do your results make sense?

9. Consider an infinite square well

$$V(x) = \begin{cases} 0 & \text{if } |x| \leq \frac{a}{2} \\ +\infty & \text{otherwise} \end{cases}$$

subject to the time-dependent perturbation

$$H_1(t) = \lambda X \sin \omega t \qquad 0 \leq t \leq T.$$

(a) Calculate and plot (as a function of ω) the transition probabilities to the first, second, and third excited states assuming the system in the ground state initially. Set

$$\omega = \frac{8\pi^2\hbar^2}{2ma^2}$$

and plot the same probabilities as a function of T. Explain the results in all cases.

(b) For the cases in (a), what is the probability that the ground state survives the perturbation?

(c) Assume now that the initial state of the system is an equal admixture of the ground and first excited state. Repeat part (a), making sure to explain the results and contrast them to those in (a).