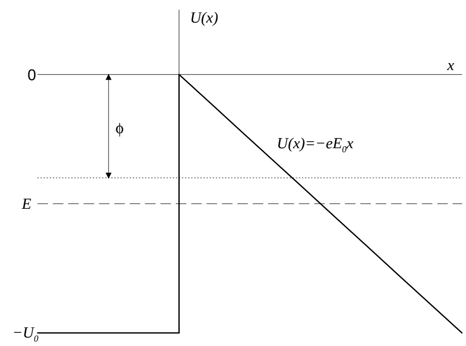
Homework 5

Due in class Oct. 8

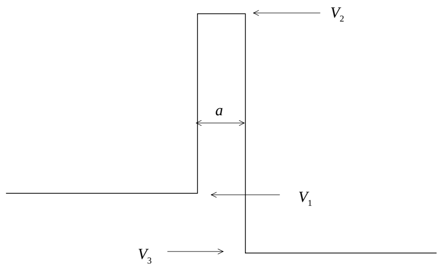
1. Field emission occurs when an electron is pulled from a metal surface by a strong electric field E_0. This process can be modeled by the potential $U(x)$ in the figure. The electron is bound inside the metal, $x < 0$, with energy E. The work function of the metal ϕ is the energy required to extract an electron at the Fermi energy E_F, *i.e.* the least amount of energy that will free an electron in the absence of an electric field. Electrons with lower energies, however, can also tunnel out.



(a) Based on qualitative arguments, what total energy do you expect most electrons will have upon leaving the metal?

(b) Quantitatively estimate the tunneling probability as a function of energy.

(c) Plot your result from (b) and briefly discuss it physically.



2. Consider a particle of mass m subjected to the potential in the figure to the left. You may assume that $V_2 - V_1 = 3\Delta V$ where $\Delta V = V_1 - V_3$.

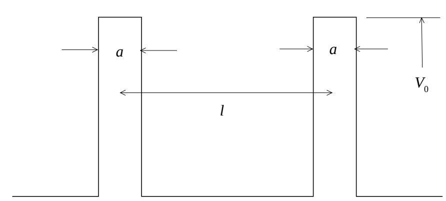
(a) Calculate the reflection and transmission coefficients as a function of energy for all energies (you should consider both incidence from the left and from the right).

(b) Calculate the tunneling probability approximately as a function of energy.

(c) Plot your results from (a) and (b), comparing the exact and approximate results. Comment on whether the agreement (or disagreement) is as you expected.

(d) Discuss physically any structure in R and T that you might have found. Give evidence for your statements.

3. Given the potential shown at the right:



(a) Calculate the reflection R and transmission T coefficients for a particle incident from the left for all energies.

(b) Calculate the phase shift ϕ for transmission, also for all energies.

(c) Plot T and ϕ and relate any features in T to features in ϕ and to the wave function. Comment on their physical interpretation.

4. Consider the following potential:

$$V(x) = V_0 \left[\left(\frac{x}{\alpha} \right)^2 - 1 \right]^2$$

where α and V_0 are constants. A potential of this form is a good model for the position of the nitrogen atom in ammonia, NH_3. The shape of ammonia can be pictured as a pyramid with a triangular base — the H's lie at the corners of the base, and the N at the top. There is no reason, however, for the nitrogen to stay above the H's, and it can tunnel through the base to be on bottom.

(a) Sketch or plot the potential and your guess for the ground and first excited state wave functions.

(b) For a particle localized on one side of the barrier, the potential appears nearly harmonic near the minimum. Find the effective harmonic potential for such a particle.

(c) Construct an approximate ground and first excited state wave function as a linear combination of the ground states of the right- and left-side simple harmonic oscillator (SHO) potentials from (c). Calculate the energies of your approximate states from the expectation value of the Hamiltonian with the full potential. Use $V_0 = \frac{6\hbar^2}{m\alpha^2}$ and $V_0 = \frac{8\hbar^2}{m\alpha^2}$. Explain the change in energies between these two values of V_0. The energies of the SHO states were the same — why are the energies of your approximate states different?

(d) Estimate the time it takes for a particle localized to the left of the barrier to appear on the right side (use the values of V_0 from (c)). (HINT: Your approximate eigenstates will evolve in time with a phase approximately given by their energies).

5. Consider a pair of square wells of width a and depth $-V_0$ that are separated by a distance l (center to center), i.e. change the barriers in the figure from Prob. 3 to wells. Such a potential is qualitatively very much like what an electron might see in a diatomic molecule (say H_2^+). The electron moves much faster than the heavy protons, so they appear essentially fixed in space.

(a) Calculate the bound state wave functions and energies as a function of l. You may want to use the program available on the class web site, but you will need to put in the potential yourself. Plot the energies as a function of l — these turn out to be effective potential curves for the motion of the protons (once the proton-proton repulsion is added). Explain the behavior of the curves as a function of l, especially the limits $l \to 0$ and $l \to \infty$. You may want to plot the wave function at a couple of representative values.

(b) Add one more potential well a distance l from its neighbor. Repeat part (a).

(c) Repeat for N=4,5, ... potential wells to deduce what happens to the energy levels for $N \to \infty$ and explain your conclusion.

Extra Credit

1. Continuing the computer problem from HW #4 ... For one of the above potentials (or one of your own choosing), numerically solve the time-dependent Schrödinger equation using the initial condition from HW #4.

(a) Plot the probability density for a few representative times.

(b) Describe the main features of the wave packet's evolution.

(c) Explain why the behavior of the wave packet is reasonable in terms of what you've learned about one-dimensional problems (i.e. relate it to R and T, etc.).

2. For the potential

$$V(x) = -V_0 a \left(\delta(x-a) + \delta(x+a) \right)$$

with a and V_0 positive, real constants,

(a) Calculate the bound state wave functions and energies and plot the wave functions. Comment on their symmetry in relation to their energy.

(b) Calculate the reflection and transmission coefficients for scattering from this potential. Plot T and discuss it.