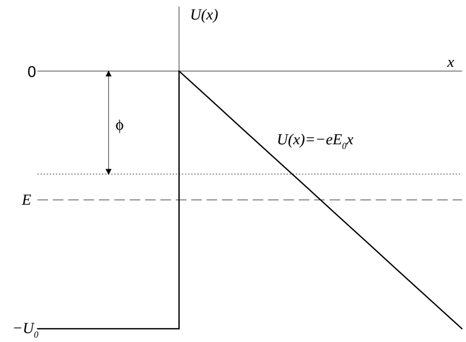
Homework 4

Due in class Sept. 24

From Shankar: Exercises 5.2.1, 5.4.2a, and



3. Field emission occurs when an electron is pulled from a metal surface by a strong electric field E_0. This process can be modeled by the potential $U(x)$ in the figure. The electron is bound inside the metal, $x < 0$, with energy E. The work function of the metal ϕ is the energy required to extract an electron at the Fermi energy E_F, *i.e.* the least amount of energy that will free an electron in the absence of an electric field. Electrons with lower energies, however, can also tunnel out.

(a) Based on qualitative arguments, what total energy do you expect most electrons will have upon leaving the metal?

(b) Quantitatively estimate the tunneling probability as a function of energy.

(c) Plot your result from (b) and briefly discuss it physically.

4. Consider a free electron whose initial wave function is given by the Gaussian wave packet of Eq. (5.1.14) on p. 154. (See also HW 3, #5.)

(a) Numerically solve the time-dependent Schrödinger equation using this initial condition and plot the probability density for several times over a range of about 20 time units (or more). Feel free to use whatever means you like for the numerical work, although simply plotting the analytical wave function is not acceptable. A simple fortran program is also available on the class web page. (Note that it uses atomic units: $\hbar$=m_e=e=1.)

(b) Describe the main features of the wave packet's evolution.

(c) Verify that your numerical wave packet behaves as you expect based on your solution to HW 3. In particular, compare $\langle X\rangle(t)$ and $\Delta X(t)$ between the two calculations. If you use the program from the class web page, you should approximate integrals as

$$\int dx\ f(x) \approx \sum_i f(x_i)\Delta x.$$

(For those of you not familiar with fortran, the code section to do the sum would be:

```
Sum = 0.0d0
do i = 1,xNumSteps
  Sum = Sum + f(i)
enddo
Sum = Sum*xDelt
```

)

(d) Add a potential step and scatter your Gaussian wave packet from it. Show several representative plots of the probability density and numerically calculate the transmission and reflection probabilities. Comment on whether the results make sense physically. Repeat for mean kinetic energies both above and below the potential step height.

Extra Credit

1. Quantitatively compare your numerical results for the transmission and reflection coefficients of the step potential to an analytical prediction (a clear description of what must be done is worth some points here, too).

2. A particle of mass m is confined to move around a ring of radius a. There is thus only one degree of freedom — the angle ϕ. The particle sees a potential given by

$$V(\phi) = \begin{cases} +V_0 & \text{if } 0 \le \phi \le \frac{\pi}{2} \\ 0 & \text{otherwise.} \end{cases}$$

What are the allowed energies for this system?