Homework 3

Due in class Sept. 17

From Shankar: Exercises 4.2.1, 4.2.2, 4.2.3, and

4. Consider a particle of mass m submitted to the potential:

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ +\infty & \text{otherwise.} \end{cases}$$

$|\phi_n\rangle$ are the eigenstates of the Hamiltonian H of the system, and their eigenvalues are

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \ldots$$

The state of the particle at the instant $t = 0$ is:

$$|\psi(0)\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle + a_3|\phi_3\rangle + a_4|\phi_4\rangle.$$

a. What is the probability, when the energy of the particle in the state $|\psi(0)\rangle$ is measured, of finding a value smaller than $\frac{3\pi^2\hbar^2}{ma^2}$?

b. What is the mean value and what is the root-mean-square deviation of the energy of the particle in the state $|\psi(0)\rangle$?

c. Calculate the state vector $|\psi(t)\rangle$ at the instant t. Do the results found in a. and b. above at the instant $t = 0$ remain valid at an arbitrary time t?

d. When the energy is measured, the result $\frac{8\pi^2\hbar^2}{ma^2}$ is found. After the measurement, what is the state of the system? What is the result if the energy is measured again?

5. Consider a free particle whose initial wave function is given by the Gaussian wave packet of Eq. (5.1.14) on p. 154.

a. Calculate the corresponding momentum space wave function and write the coordinate space wave function as an expansion on momentum space basis functions.

b. Verify explicitly the time-dependent wave function in Eq. (5.1.15).

c. Verify explicitly the expressions in the text for $\langle X\rangle(t)$ and $\Delta X(t)$.

Supplemental reading:

Skim the Examples in Chap. 4 to make sure that you understand them.