Homework 11

Due in class Dec. 5

1. Given $\ell_1 = 1$ and $\ell_2 = 5$ (note that you may calculate any Clebsch-Gordan coefficients by whatever method you like):

(a) Calculate the possible values of the total angular momentum $\mathbf{L} = \mathbf{L_1} + \mathbf{L_2}$.

(b) Write down the explicit expressions in terms of $|\ell_1 m_1\rangle|\ell_2 m_2\rangle$ for the states

(*i*) $|(15)3-3\rangle$

(*ii*) $|(15)4-3\rangle$

(*iii*) $|(15)6-6\rangle$

(*iv*) $|(15)65\rangle$.

(c) Turn the expansion around and write the state $|11\rangle|5-4\rangle$ as an expansion on eigenstates of total angular momentum.

2. Consider a hydrogen atom in a constant external electric field $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$.

(a) Calculate the correction to the ground state energy in first order perturbation theory.

(b) Calculate the corrections to the $n = 2$ states in first order perturbation theory.

(c) For the $m = 0$ states, argue that the direction of the energy shift relative to the unperturbed energies makes sense. (A good starting point is the electron density described by the wave functions.)

(d) Sketch the energies you obtained in (a) and (b) as a function of the magnitude of the electric field $\mathcal{E}$. For what values of $\mathcal{E}$ are these results valid?

The shifts you calculated are called the Stark effect; and the states, the Stark states. (HINTS: Write z in terms of Y_{lm}'s and notice that

$$\int d\Omega\ Y_{00}^* Y_{lm} Y_{l'm'} = \frac{1}{\sqrt{4\pi}} \int d\Omega\ Y_{lm} Y_{l'm'}$$

since $Y_{00} = 1/\sqrt{4\pi}$.)

3. Consider the spin-orbit interaction in the hydrogen atom,

$$W = \frac{e^2}{2m^2c^2} \frac{\mathbf{L}\cdot\mathbf{S}}{r^3}$$

where e and m are the charge and mass of the electron, $\mathbf{L}$ is its orbital angular momentum, $\mathbf{S}$ is its spin, and r is its distance from the proton.

(a) Calculate the first-order energy shift of the ground state.

(b) Calculate the first-order energy shifts of the $n = 2$ states.

This correction is one of the relativistic corrections to the simple hydrogen energy spectrum that you're used to. (HINT: Notice that $J^2 = L^2 + S^2 + 2\mathbf{L}\cdot\mathbf{S}$.)

4. If we take the e-e interaction to be a perturbation in the helium atom, then the zero-th order eigenstates are just hydrogenic solutions with $Z = 2$ when spin-orbit and higher-order interactions are neglected.

(a) Write down the ground singlet and triplet states (properly symmetrized and angular momentum eigenstates!). Argue that orbital and spin angular momenta are separately conserved.

(b) Calculate to first order the shifts to the states in (a) due to the electron-electron interaction. Do the directions of the shifts make sense? (HINT: Consider the two-electron density.)

(c) Repeat (a) and (b) for the H^- ion. Is H^- stable in this approximation, or is it more favorable to break up into H+e?

5. Using Fermi's Golden Rule, calculate the $1s \to 2\ell$ transition rates for hydrogen induced by linearly polarized light. In general, which transitions can be induced by linearly polarized light starting from the ground state?