Homework 10

Due in class Nov. 26

From Shankar: Exercises 10.3.2, 10.3.3, 10.3.4, 12.3.3, 12.5.3, 12.5.12, and

7. Consider two identical particles of mass m in an infinite square well of width a (see exercise 10.3.4).

(a) Assuming the particles are *bosons*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

(b) Assuming the particles are *fermions*, write down the wave functions for the lowest three energy states of the system. What are their degeneracies?

(c) Choose values for the parameters (one set only) and plot the wave functions you obtained in (a) and (b) — this will be a function of two variables, so you need access to a program to plot a surface (or contour). Mathematica can handle this and is available on the departmental computers. Comment on any general patterns you find.

8. Consider two identical particles of mass m in an infinite square well of width a (see exercises 8 and 10.3.4). Now, add the interparticle interaction

$$H_1 = \frac{\lambda}{a^2}(x_1 - x_2)^2$$

as a perturbation.

(a) Assuming the particles are *bosons*, calculate the first-order shift in the energies of the two lowest energy states.

(b) Assuming the particles are *fermions*, calculate the first-order shift in the energies of the two lowest energy states.

(c) Where possible, compare the shifts in (a) and (b) and discuss the physical basis of their relative magnitude (in terms of the wave function, for instance).

9. Consider a particle in a state described by

$$\psi(\mathbf{x}) = N(x + ye^{i\frac{\pi}{4}} - z + 1)e^{-\alpha r}$$

in spherical coordinates where N is a normalization factor and α is a positive, real constant. Following the general method outlined in exercise 12.5.13, calculate:

(a) The probability that a measurement will yield $\ell = 0, 1, 2, \ldots$

(b) The probability that a measurement will yield $m = \ldots, -1, 0, 1, \ldots$.

(c) The probability that a measurement will yield $\ell = 1$ and $m = -1$.

10. Find the lowest three energy levels of a particle in an infinite spherical box of radius a for $\ell = 0$ and for $\ell = 1$.

11. Consider a spin-$\frac{1}{2}$ particle placed in a magnetic field $\mathbf{B}$:

$$\mathbf{B} = \frac{B_0}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{z}}).$$

The Hamiltonian for this system is given by

$$H = -\mathbf{M}\cdot\mathbf{B} = -\gamma\mathbf{S}\cdot\mathbf{B}$$

where $\mathbf{M} = \gamma\mathbf{S}$ is the magnetic moment of the particle, γ is a constant, and $\mathbf{S}$ is the spin angular momentum operator.

(a) Calculate the matrix representing H in the $\{|+\rangle, |-\rangle\}$ basis.

(b) Calculate the eigenvalues and eigenvectors of H.

(c) At $t = 0$, the system is in the state $|-\rangle$. Write down the state vector $|\psi(t)\rangle$ at an arbitrary time t later. If S_x is measured, what is the mean value of the results that can be obtained?

Shankar 14.4.3: This problem details the essential quantum mechanics of NMR, and shares many characteristics with a two-level system drive by a laser of frequency ω. Derive Eq. 14.4.36, then answer:

(a) What is the probability that at some time t a measurement of the spin will yield $s_z = \frac{\hbar}{2}$?

(b) Calculate $\langle\mu_z(t)\rangle$ and verify that it agrees with Eq. 14.4.31.

(c) Discuss what happens when $\omega_0 = \omega$ — both physically and mathematically.