Homework 1

Due January 25, beginning of class

1. Two particles with reduced mass μ scatter from each other via a finite spherical square well interaction. The square well has radius r_0 and depth $-V_0$ ($V_0 > 0$). Consider potential strengths of $\sqrt{2\mu r_0^2 V_0}$ of 5.8, 5.9, 6.0, 6.1, 6.2, and 6.3.

(a) Verify Levinson's Theorem for $\ell = 0, 1$ in each case.

(b) Calculate the scattering length a in each case.

(c) For the two extreme potential strengths, plot $\delta_{\ell=0}$ for scattering energies from zero to "∞". Justify your choice of ∞ physically.

(d) Calculate the total cross section σ in each case. Verify that σ is converged to 1% accuracy for energies below $9/(2\mu r_0^2)$.

(e) Explain any features you found in σ physically. In particular, interpret the evolution of each feature with potential strength. It will probably be useful to show the phase shifts and/or partial cross sections for this purpose.

(f) Pick one potential strength and verify that each partial wave exhibits the expected Wigner threshold behavior.

2. One model for $e^- + \mathrm{H}(1s)$ elastic scattering uses the following static interaction potential:

$$V(r) = -\frac{e^{-2r}}{r}(1 + r).$$

(a) For what scattering energies is this model most appropriate? That is, when will it definitely break down?

(b) Calculate the total scattering cross section for this model over the energy range you stated in (a).

(c) Interpret the cross section physically.