Exam 1: Theory of Atomic Structure

Due Oct. 2, 2000

You may use any resource you like to complete this exam including computers, Mathematica, integral tables, and the text. If you do, make sure you cite that resource. You are, however, to complete this exam on your own. Any plots should be computer generated.

This exam is due promptly at the beginning of class Monday, Oct. 2. No exceptions will be made.

1. Calculate the splitting of the $n = 2$ level of hydrogen due to an external magnetic field $\mathbf{B} = B\hat{z}$ as a continuous function of B. Plot the energy levels over the whole range of B from weak to strong field limits. On the plot, identify the regimes of the anomolous Zeeman effect, the normal Zeeman effect, and the Paschen-Back effect. If you include the diamagnetic term, indicate its contribution on the plot. If you do not include it, justify its omission physically. Based upon your wave functions, identify the approximately good quantum numbers in each regime (*i.e.* use the coefficients of your basis functions).

2. Consider the motion of a particle around a circle, experiencing the potential

$$V(\phi) = V_0 \cos\phi.$$

   a. Identify any symmetriex of this system.

   b. Using an appropriate basis set and the variational principle, calculate the ground and first excited state energies to a relative accuracy of at least 1%.

   c. Explain the meaning of the Hylleraas-Undheim theorem in your own words and use your calculation from b. to demonstrate its validity.

3. Calculate the radiative lifetimes of the $3s$, $3p$, and $3d$ states of hydrogen in the dipole approximation.

4. By some combination of fields, a helium ion — $He^+(1s)$ — is trapped by the potential

$$V = \frac{1}{2} M\omega_0^2 R^2$$

   where M is the total mass of the ion and R is the distance from the center of the trap to the center of mass of the ion. These external fields have no other effect. An absorption spectrum is generated by exposing the ion to a tunable laser. Quantitatively describe the spectrum as a function of the laser frequency ω in the dipole approximation for the three cases:

   a. $\omega_0 = \frac{1}{10}\omega_{21}$

   b. $\omega_0 = \omega_{21}$

   c. $\omega_0 = 10\omega_{21}$.

   ω_{21} is the $n = 2$ to $n = 1$ Bohr frequency for He^+.

Bonus Question. A photon carries linear momentum as well as angular momentum. When an electronic transition is induced in a free hydrogen atom, what happens to the linear momentum of the photon? Your answer should be quantitative and quantum mechanical.