

Homework 2

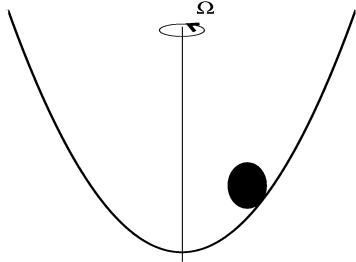
Due at the beginning of class Wednesday, Feb. 1

From the text: 2-14, 2-20, 3-13

1. Consider a central potential of the form $V(r) = -k/r^2$, $k > 0$.

- (a) Sketch the effective potential for angular momentum ℓ such that $\ell^2/2m > k$, $\ell^2/2m = k$, and $\ell^2/2m < k$.
- (b) Identify the possible kinds of orbits for each case in (a).
- (c) Explicitly calculate the orbits you identified in (b). Plot them and discuss them physically.

2. As shown in the figure below, a vertical parabolic track is constructed so that an object can roll along its inside surface without slipping. With some external force, the track is rotated at a constant angular frequency Ω about the vertical axis as indicated. The rolling object always stays in the same plane as the track.



In the plane of the track, with the vertical direction labeled y and the horizontal x , the equation describing the shape of the track is $y = \frac{1}{2}x^2$. The system is subject to gravity.

- (a) Assuming that the rolling object has mass m , radius r , and moment of inertia I , obtain the equations of motion using the Lagrangian formalism. Use any first integrals to simplify your equations of motion. Identify any such quantities physically.
- (b) Identify an effective potential and sketch it. Discuss it physically. In your discussion, be sure to calculate the equilibrium position(s) and discuss whether they are stable or not. Also, be sure to indicate how they depend on the parameters of the problem (m , r , I , Ω) and if there are physically significant features associated with this dependence.