Homework 9

Due in class Monday, April 11

From the text: 10-13, 10-18

1. Consider a pendulum consisting of a mass m attached to the end of a rigid rod of length ℓ. Take the rod to be massless but free to rotate in a vertical plane. The system is, of course, subject to gravity.

(a) Discuss the system trajectories in phase space under all possible initial conditions, noting especially the transition between different types of motion, turning points, etc. To aide your discussion, plot H in phase space at several representative energies.

(b) Using action-angle variables, find the frequency of oscillation of the system for the cases you discussed in (a). Do **not** make the small-angle approximation.

(c) Discuss your result from (b) physically (a plot might be helpful). Make sure to discuss its connection to your phase space trajectories from (a).

2. Let us now consider a somewhat realistic model for a molecular potential:

$$V(x) = V_0 \left[\left(\frac{x}{\alpha} \right)^2 - 1 \right]^2$$

where α and V_0 are positive, real constants. A potential of this form is a good model for the position x of the nitrogen atom in ammonia, NH_3, relative to the plane of the hydrogens (so x is simply a Cartesian coordinate). The shape of ammonia can be pictured as a pyramid with a triangular base — the H's lie at the corners of the base, and the N at the top. Quantum mechanically, there is no reason for the nitrogen to stay above the Hs, and it can tunnel through the base to be on bottom. This leads to a splitting of energy levels that has been used to produce a maser.

But, we'll consider this potential classically...

Using action-angle variables, obtain the frequencies of oscillation for the system under all possible initial conditions in terms of the various parameters in the problem. Discuss your answer physically.