Homework 5

Due in class Monday, March 7

From the text: 8-2, 8-26

1. Two identical pendula are hung from the same point. Each pendulum consists of a massless rod of length ℓ with a point charge on the end. Each charge q has mass m. The pendula move only in the same plane.

(a) Write down the Lagrangian for the system.

(b) The charges of the particles are such that the two masses hang a distance $d = \ell/2$ apart when the pendula are at rest. Calculate the charges q.

(c) Find the frequencies of small oscillations about equilibrium and the associated normal modes. Describe them physically.

(d) At t=0, one pendulum is pushed away from the other such that its charge is displaced $\ell/20$ from equilibrium. The other pendulum is kept at its equilibrium. Write the expressions for the subsequent motion of the pendula. Plot their motion as a function of time. Be sure to extend the plot to long enough times to clearly show any pattern if a pattern is present. And, if there is a pattern, discuss it physically.

2. When I was a graduate student, I was asked to calculate the normal modes for identical ions restricted to move only along a line and confined by a harmonic oscillator trap. Knowing these normal modes is important for quantum computing with trapped ions. The potential for this system is (in atomic units)

$$V = \sum_{i=1}^{N} \frac{1}{2} m\omega^2 x_i^2 + \sum_{i<j}^{N} \frac{q^2}{|x_i - x_j|}$$

where the x_i refer to some lab-fixed coordinate system. Note that you may need to evaluate some quantities numerically.

(a) For two ions (N=2), find the frequencies of small oscillations and the corresponding normal modes. Discuss the normal modes physically. Be sure to indicate what "small" is.

(b) Repeat (a) for N=3. Additionally, plot the eigenvectors — that is, take each element of the eigenvector as the y value and the index as the x value for plotting. Discuss these plots, comparing their behavior and correlating it to their eigenfrequency.

(c) Repeat (b) for N=4.

(d) Can you predict what happens to the eigenfrequencies as N increases? To the eigenmodes? Your answer should include sketches...

(e) For N=3, at t=0 the middle ion is displaced in the positive direction by 10% of the distance to the next ion's equilibrium. Both of the other ions remain at their equilibrium. Find the subsequent motion of all three ions. Is there ever a time when the system returns to this configuration?

(f) For N=3, at t=0 a uniform electric field is turned quickly on and off. The field points along the ions' direction of motion, and is on for such a short time that it merely gives all of the ions an initial velocity (their displacement is negligible). Assuming that the magnitude of this initial velocity is v, find their subsequent motion. Discuss it physically.