Homework 12

Due in class Monday, May 2

1. The modes of a clamped membrane whose boundary is an isosceles right triangle are identical with those of a clamped square membrane with a diagonal nodal line. Find the lowest three eigenfrequencies and plot their displacements. A very similar approach can be used to find the eigenfrequencies of an equilateral triangular membrane.

2. A string of uniform mass density σ and length ℓ hangs under its own weight. Consider small transverse displacements in a plane.

(a) Compute the equilibrium tension in the string $\tau(x)$, where x is the distance from the point of suspension.

(b) Show that the normal modes satisfy Bessel's equation. (HINT: try the substitution $s^2 = \ell - x$.)

(c) What are the boundary conditions?

(d) What condition do the normal-mode frequencies satisfy? Explicitly find the lowest three eigenfrequencies.

(e) What are the normal modes? Plot the lowest three. Discuss them physically. Do they make sense?

(f) If the lower end of the string is initially displaced $\ell/100$ from equilibrium such that the string remains straight, what is the subsequent motion of the string? Your answer can contain integrals. Evaluating all necessary integrals and discussing the mode distribution of the motion is worth some extra credit...

EC1. Consider a clamped circular membrane of radius a whose tension and areal mass density are constant.

(a) Find the condition(s) that the eigenfrequencies satisfy. Explicitly find the lowest three eigenfrequencies. Be sure to discuss any degeneracies.

(b) Find the normal modes. Plot the lowest three. Discuss them physically. Do they make sense?

(c) At $t = 0$, a stone of mass m is dropped in the exact middle of the membrane from a height of $3a$. Find the subsequent motion of the membrane. Be sure to discuss the mode distribution of the motion.

(d)

EC2. A clamped circular membrane of radius a has uniform areal mass density. It lies horizontally and thus experiences gravity.

(a) Find the wave equation satisfied by the membrane.

(b) Solve your wave equation and explicitly find the lowest three eigenmodes (frequencies and displacements). Plot these three displacements. Discuss the effect of gravity on these eigenmodes? For instance, does gravity increase or decrease the eigenfrequencies?