Homework 11

Due in class Monday, April 25

1. A uniform string under tension τ with fixed endpoints $\rho(0) = \rho(\ell) = 0$ is plucked in the middle, giving rise to an initial velocity distribution

$$\dot{u}(x,0) = v_0 \ell \delta(x - \frac{1}{2}\ell).$$

Find u(x,t) and calculate the energy in each mode.

2. A uniform string with fixed endpoints has length ℓ , mass density σ and uniform tension τ . A point mass m is attached at its center. Show that the modes in which m moves have frequencies that satisfy the equation

$$\frac{2c}{\omega\ell}\cot\frac{\omega\ell}{2c} = \frac{m}{\sigma\ell}$$

where $c^2 = \tau/\sigma$. Discuss the solutions in the limiting cases $m \to \infty$ and $m \to 0$.