Advanced Dynamics
Exam 2

Monday, April 18, 2011

Instructions:

- The exam is worth a total of 100 points, with the points for each problem indicated.
- If a question is unclear, ask.
- Cite anything you didn't do yourself — from notes, books, tables, etc.
- Start each problem on a new sheet of paper.
- Remember to write down everything you can about a problem — including a figure! Even if you can't find the solution because of time or difficulty, you can write down what you think needs to be done and what physics you expect to result.
- Your answer should be written clearly, logically, and legibly.
- Any plots, discussion, or physical insight beyond what is asked for has a good chance of becoming extra credit ...

(20 pts) 1. Briefly answer the following:

(a) To what kinds of systems do action-angle variables apply and how are action-angle variables useful for these systems?

(b) List at least four ways to define or verify that variables $Q = Q(q, p)$ and $P = P(q, p)$ are canonically conjugate.

(c) Describe at least three close connections between the classical mechanics we have discussed since the last exam (not including continuum mechanics) and quantum mechanics. This should not simply be a list. Rather, it should also include a brief description of the physics involved that makes it clear that you understand the connection.

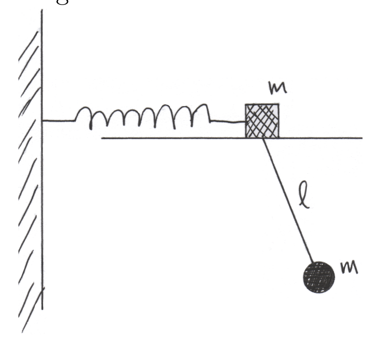
(d) From the material we have covered since the last exam, what do you find to be the most surprising or interesting result (homework problem, derivation, connection with other fields, etc.) and why? You will be graded on how well you communicate the physics context of the result and on how well you communicate why you found it interesting/surprising.

(20 pts) 2. Using action-angle variables, find the frequency of oscillation as a function of energy for any periodic motion there might be in the following potential:

$$V(x) = \begin{cases} \frac{1}{2}kx^2 & x \le 0 \\ \alpha x & 0 \le x \le x_0 \\ \alpha x_0 & x \ge x_0 \end{cases}$$

where k, x_0, and α are positive, real constants. Do not make any approximations. Sketch your frequency and discuss it physically.

(20 pts) 3. In the system below, the square mass moves freely (*i.e.* no friction) along the horizontal surface as indicated. It can only move in the direction perpendicular to the wall. It is attached to the wall by a spring with spring constant k and relaxed length a. Hanging from this mass is a pendulum that is free to move in the same plane as the square-mass-spring system. The pendulum consists of a massless rod of length ℓ and a mass m attached at its end.



(a) Find the frequencies of small oscillation.

(b) Find the normal modes of the system. Discuss them physically.

(c) At $t = 0$, the square mass is at rest at its equilibrium. The pendulum, however, is displaced to the right by 0.01 rad. Find the motion of the system for all later times. Are there any conditions under which the system will return exactly to its initial configuration? If so, at what time(s) does it return?

(20 pts) 4. Consider the system described by the following kinetic T and potential V energies:

$$T = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2)(q_1 + q_2) \qquad V = (q_1 + q_2)^{-1}$$

where q_1 and q_2 are generalized coordinates.

(a) What is the Hamilton-Jacobi equation for this system?

(b) Solve your Hamilton-Jacobi equation from (a) to find Hamilton's principal function.

(c) Deduce the dynamical motion of the system.

(20 pts) 5. Consider the following transformation:

$$x = \frac{1}{\alpha}\left(\sqrt{2P_1}\sin Q_1 + P_2\right)$$
$$y = \frac{1}{\alpha}\left(\sqrt{2P_1}\cos Q_1 + Q_2\right)$$
$$p_x = \frac{\alpha}{2}\left(\sqrt{2P_1}\cos Q_1 - Q_2\right)$$
$$p_y = -\frac{\alpha}{2}\left(\sqrt{2P_1}\sin Q_1 - P_2\right).$$

(a) By any method you choose, show that the transformation is canonical.

(b) Find a generating function of type F_3 that produces this transformation.