AQM-2013f Quiz II, October 17, 2013 2:30-3:45pm

Open Book

1. Let

$$|\Psi >= \sqrt{1/3}\, Y_{10}\, \chi_+ \; + \sqrt{2/3}\;\; Y_{11}\, \chi_-$$

The χ's are spin functions for s=1/2.

(a) If you measure S_z , what is the probability that you will get ħ/2?

(b) If indeed you do get ħ/2 in (a), what is the new state (or wavefunction) after the measurement?

(c) If you measure L_z, after (b), what is the probability that you will get –ħ?

(d) Start over. If you measure the J_z of the original state $|\Psi >$, what are the possible value(s) you will get? (20 pts).

2. (a) Sketch the hydrogenic <u>radial</u> wavefunctions 1s, 2p, 3d. Try to have the correct scale.

(b) Sketch the radial wavefunctions of a spherical infinite square well, for the first state of each ℓ=0,1,2. Try to be as accurate as you can. The radius of the sphere is a. (20 pts)

3. (a) A muonium is consisting of a μ^+ and a μ^-. Calculate the transition energy (in keV) between the lowest and the second lowest states. (10 pts)

(b) Calculate $< Y_{21} \,|\, L_x \,|\, Y_{20} >$ and $< Y_{20} \,|\, L_x \,|\, Y_{20} >$. (10 pts)

4. For the hydrogenic 3d state with the inclusion of spin,

(a) What is the degeneracy of the 3d level if spin is included? (5pts)

(b) The coupling of orbital and spin angular momenta would give new eigenstates of J^2 and J_z, with quantum numbers j, m, respectively. What are the possible values of j? what is the degeneracy for each j? Show that the total degeneracy agrees with the one obtained in (a). (15 pts)

(c) The j levels will split due to spin-orbit interaction which can be written as A $\vec{\ell} \cdot \vec{s}$. Calculate the expectation value of this term for the different j's you have. (10 pts)

5. For an isotropic harmonic oscillator with the potential

$$V(r) = \frac{1}{2} m\omega^2 r^2 ,$$

the wavefunction can be expressed in spherical coordinates

$$\psi(r,\theta,\phi) = R(r) Y_{\ell m}(\theta,\phi)$$

Consider the states with $\ell = 2$ and m=0. What is the energy of the lowest state and its radial wavefunction R(r) for this set of $\ell = 2$ and m=0 ?

[Note: This is a more challenging problem, but you should be able to use the known harmonic oscillator wavefunctions in Cartesian coordinates to find the answer. This is an extension from the homework assignment.] (8 pts for the energy and 12 pts for R(r)]