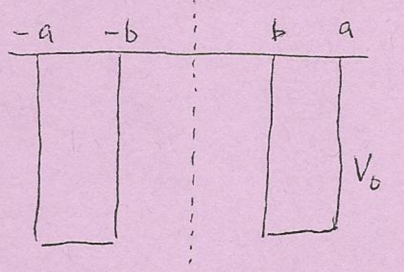
AQM-2013f Quiz I September 19, 2003 2:30pm-3:45pm

Open book

1. For the one-dimensional harmonic oscillator, let |n> be the energy eigenstates (n=0, 1,2, 3, ..) . Any operator in the {|n>} basis set is represented by a matrix.
   (a) Express the creation operator a_+ in matrix form and write down explicitly the first 4x4 matrix elements.
   (b) Do the same for the operator x. (20 points, 10 each)

2. Sketch the energies and eigenfunctions of the first three lowest states of the 1D potential below. (DO NOT COMPUTE). Draw them to the scale as accurately as you can, including the relative spacing of the energy levels and the key properties of the wavefunctions.
   (20 points, 8 points for energy, and 4 points for each wavefunction)



3. For S=1, a state is represented by

$$U=\frac{1}{\sqrt{26}}\begin{pmatrix}1\\4\\-3\end{pmatrix}$$

   What is the probability that a measurement of S_x yields the value 0? (15 points)

4. A particle is known to be localized in the left half of a box with equal probability between x=0 and x=a/2. The two sides of the box are at x=0 and x=a.
   (a) Find the normalized initial wavefunction at t=0.
   (b) What is the wavefunction of the particle at later time t? Express it in general form vs time t?
   (c) In an energy measurement, what is the probability of finding the particle in the ground state, and in the first excited state? Do the probabilites change with time? (30 points, 10 each)

5. For an infinite square well potential, with boundaries at x=0 and x=a, if a delta potential $\alpha\,\delta(x-a/2)$ is added at the half way point x=a/2, show that the ground state energy will be shifted, but the energy of the first excited state will not change. Here α is a positive constant.
   [set up the equations to explain your results, but no need to solve them.) (15 points)