Chapter 6c: perturbation Theory
Hydrogen in electric or magnetic fields

(1) The Zeeman effect — weak field

$$H'_Z = -(\vec{\mu}_\ell + \vec{\mu}_s)\cdot\vec{B}$$

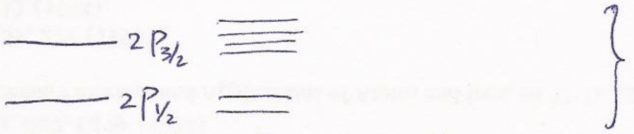
$$\vec{\mu}_\ell = -\frac{e}{2m}\vec{L}, \quad \vec{\mu}_s = -\frac{e}{m}\vec{S}$$

where the dipole moment due to the orbiting electron is $\vec{\mu}_\ell$ and due to the spin is $\vec{\mu}_s$.

Let $\vec{B} = B\hat{z}$

$$H'_{Z} = \frac{e}{2m}(\vec{L} + 2\vec{S})\cdot\vec{B} = \frac{eB}{2m}(L_z + 2S_z)$$

We will take the 2p state of hydrogen as an example. Without spin-orbit interaction, there are six degenerate states. With the inclusion of spin-orbit interaction, it splits into



6 levels

This subscript is J

If the B field is weak

If the magnetic field is small such that the splitting is small compared to the $2P_{1/2}$ and $2P_{3/2}$ splitting, then the unperturbed Hamiltonian is

$$H = H_0 + H_{SO}$$

the zero-order wavefunctions are $|n\ell j m\rangle$ states

Apply the 1st-order perturbation, need to calculate

$$\langle jm|H_Z|jm\rangle.$$

Let us calculate the matrix elements for $2P_{1/2}$

$$E'_m = \frac{eB\hbar}{2m} \langle jm | L_z + 2S_z | jm \rangle$$

Consider $|jm\rangle = |\tfrac12\ \tfrac12\rangle$

Find C-G coefficients

$$|\tfrac12\ \tfrac12\rangle = \sqrt{\tfrac23}\ |11\rangle|\tfrac12\ -\tfrac12\rangle - \sqrt{\tfrac13}\ |10\rangle|\tfrac12\ \tfrac12\rangle$$

$$(L_z + 2S_z)|\tfrac12\ \tfrac12\rangle = \sqrt{\tfrac23}\ \cancel{(1+2\times(-\tfrac12))}\ |11\rangle|\tfrac12\ -\tfrac12\rangle + \left(-\sqrt{\tfrac13}\right)\left(0 + 2\times\tfrac12\right)|10\rangle|\tfrac12\ \tfrac12\rangle$$

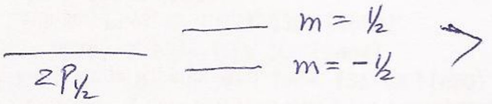
$$= -\sqrt{\tfrac13}\ |10\rangle|\tfrac12\ \tfrac12\rangle$$

Thus

$$\langle \tfrac12\ \tfrac12 | L_z + 2S_z | \tfrac12\ \tfrac12 \rangle = \frac13$$

Consider $|jm\rangle = |\tfrac12\ -\tfrac12\rangle = -\sqrt{\tfrac23}\ |1\,-1\rangle|\tfrac12\ \tfrac12\rangle + \sqrt{\tfrac13}\ |10\rangle|\tfrac12\ -\tfrac12\rangle$

$$\langle \tfrac12\ -\tfrac12 | L_z + 2S_z | \tfrac12\ \tfrac12 \rangle = -\frac13$$



Similarly

← Zeeman splitting

(2) Zeeman effect – strong field

If the magnetic field is stronger than the effect due to spin-orbit interaction, then start with uncoupled states.

i.e., $|2p\, m_\ell\, m_s\rangle$ states

The perturbation is calculated from

$$\frac{eB}{2m}\langle 2p\, m_\ell\, m_s|\, L_z + 2S_z\,|2p\, m_\ell\, m_s\rangle$$

$$= \frac{eB\hbar}{2m}\left(m_\ell + 2m_s\right)$$

m_ℓ	m_s	$m_\ell + 2m_s$
1	$\frac{1}{2}$	2
0	$\frac{1}{2}$	1
-1	$\frac{1}{2}$	0
1	$-\frac{1}{2}$	0
0	$-\frac{1}{2}$	-1
-1	$-\frac{1}{2}$	-2

(3) Stark effect

$$H' = eEz \quad \text{where } \vec{E} = E\hat{z}$$

$$\left(\text{Force } F = (-e)E \qquad F = -\frac{dV}{dz} = (-e)E \qquad V = eEz\right)$$

$n=1$ $\quad \langle 1s|\, eEz\,|1s\rangle = 0$ by parity

no first-order perturbation term

Need to calculate the 2nd-order term

$$E^{(2)}_{1s} = \sum_n \frac{|\langle n|eEz|0\rangle|^2}{E_{1s} - E_n} \simeq E^2$$

or $= -\frac{1}{2}\alpha E^2$

↑ dipole polarizability

(3) Stark effect for $n=2$

$$H' = eEz$$

↑ contains no spin

so spin function does not enter

degenerate states

$$\varphi_{200},\ \varphi_{210},\ \varphi_{21-1},\ \varphi_{21,1}$$

4×4 matrix

$$\langle \varphi_{200} | z | \varphi_{211} \rangle = 0\ ?$$

This is in your homework.

It will end up that you need to evaluate only <u>one</u> matrix element, and diagonalize a 2×2 matrix