Quiz II Keys

(1)

20

(a) $\frac{1}{3}$.

(b) $Y_{10}\chi_+$.

(c) 0

(d) $J_z = l_z + S_z$

$$J_z|\psi\rangle = (l_z + S_z)\left(\sqrt{\tfrac{1}{3}}\,Y_{10}\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_{11}\chi_-\right)$$

$$= \sqrt{\tfrac{1}{3}}\left[0 + \left(\tfrac{\hbar}{2}\right)Y_{10}\chi_+\right] + \sqrt{\tfrac{2}{3}}\left[\hbar - \tfrac{\hbar}{2}\right]Y_{11}\chi_-$$

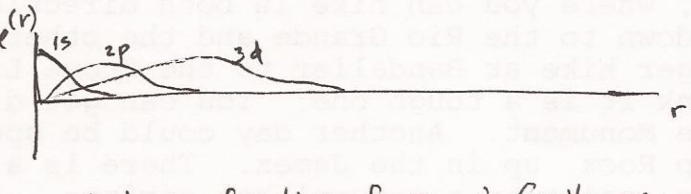
$$= \frac{\hbar}{2}\left[\sqrt{\tfrac{1}{3}}\,Y_{10}\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_{11}\chi_-\right]$$

$$= \frac{\hbar}{2}\,|\psi\rangle$$

You always get $\hbar/2$ if measured by J_z

(2)

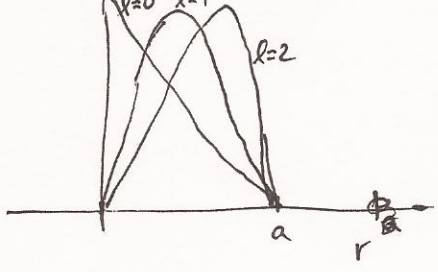
10 (a)



10 (b) The radial wavefunctions of an infinite square well are given by $j_l(k_{nl} r)$ where $k_{nl} a = \beta_{nl}$

β_{nl} is the nth zero of $j_l(x) = 0$

From Fig. 4.2 of your textbook (P. 143)



No radial node for the 1st state of each l

(3)

(a) $m_\mu = 206\ m_e$

The reduced mass is $103\ m_e$

$n=2 \rightarrow n=1$ transition energy $= 13.6\,\text{eV}\left(1-\frac{1}{4}\right)$
for H

$= 10.2\ \text{eV}$

The binding energy is proportional to the reduced mass

Thus for $n=2\rightarrow 1$ transition for $\mu^+\mu^-$ is

$10.2\,\text{eV} \times 103$

$= 1050.6\ \text{eV} = \underline{1.05\ \text{KeV}}$

(b) $L_x = \frac{1}{2}\left(L_+ + L_-\right)$

use $L_\pm\,|lm\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\ |l\ m\pm1\rangle$ $\leftarrow$ P.166 eq. [4.121] and homework

$L_+ Y_{20} = \hbar\sqrt{6}\ Y_{21}$

$L_- Y_{20} = \hbar\sqrt{6}\ Y_{2-1}$

Thus $\langle Y_{21}|\,L_x\,|Y_{20}\rangle = \hbar\frac{1}{2}\sqrt{6} = \hbar\sqrt{3/2}$

$\langle Y_{22}|\,L_x\,|Y_{20}\rangle = 0$

(4) (a) $(2l+1)\times 2 = 10$

(b) $j = |l-s| \ldots (l+s)$

$= 3/2,\ 5/2$

$j = 3/2$ degeneracy $= (2j+1) = 4$

$j = 5/2$ $= 6$

$4+6 = 10$ as in (a)

(c) Need to calculate $\langle jm|\,A\,\vec{l}\cdot\vec{s}\,|jm\rangle$

By definition

$$\vec{j} = \vec{l} + \vec{s}$$

$$j^2 = l^2 + s^2 + 2\,\vec{l}\cdot\vec{s}$$

$$\vec{l}\cdot\vec{s} = \frac{1}{2}\left[\,j^2 - l^2 - s^2\,\right]$$

since $|jm\rangle$ are eigenstates of j^2, l^2, s^2

$$A\,\langle jm|\vec{l}\cdot\vec{s}|jm\rangle = \cancel{\frac{A}{2}\left[j^2 - l^2 - s^2\right]}$$

$$= \frac{A}{2}\left\{\langle jm|j^2 - l^2 - s^2|jm\rangle\right\}$$

$$= \frac{A}{2}\left\{j(j+1) - l(l+1) - s(s+1)\right\}\hbar^2$$

For $j = 5/2 \Rightarrow \frac{A}{2}\cdot 2\hbar^2 = \hbar^2 A$

$j = 3/2 \Rightarrow \frac{A}{2}\cdot(-3\hbar^2) = -\frac{3}{2}\hbar^2 A$

(5) For $l = 2$, there are five-fold degeneracy.
Thus we need to find a level from (n_x, n_y, n_z) that gives at least having a degeneracy higher than 5

In (n_x, n_y, n_z), $E = \hbar\omega(n_x + n_y + n_z + 3/2)$

$(0, 0, 0)$ $\quad E = \hbar\omega(3/2)$ $\quad$ deg. = 1

$$\left.\begin{matrix}(1,0,0)\\(0,1,0)\\(0,0,1)\end{matrix}\right\} \quad \hbar\omega(5/2) \quad \text{deg.} = 3$$

$$\left.\begin{matrix}(2,0,0)\\(0,2,0)\\(0,0,2)\\(1,1,0)\\(0,1,1)\\(1\ 0\ 1)\end{matrix}\right\} \quad \hbar\omega(7/2) \quad \text{degen} = 6$$

Thus the $R(r)\,Y_{2m}(\theta,\phi)$ could come from the last combination

The energy then is $\underline{\frac{7}{2}\hbar\omega}$

(b) The 1D harmonic oscillator w.f. is given by

$\boxed{\text{Challenging}}$

$$\varphi_n(x) = A_n\, e^{-\xi^2/2} H_n(\xi)$$ eq. [2.85] P.56

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x = k\,x$$

$$H_2(\xi) = 4\xi^2 - 2$$

$$Y_{20}(\theta,\phi) = \frac{3}{2}\cos^2\theta - \frac{1}{2}$$

$$r^2\,Y_{20} = \frac{3}{2}z^2 - \frac{1}{2}r^2$$

$$= \frac{3}{2}z^2 - \frac{1}{2}(x^2+y^2+z^2)$$

$$= -\frac{1}{2}x^2 - \frac{1}{2}y^2 + z^2$$

Thus $R(r)\,Y_{20} = \frac{R(r)}{r^2}\left[-\frac{1}{2}x^2 - \frac{1}{2}y^2 + z^2\right]$

$\Rightarrow$ From which one can see $R(r)$ is a linear combination:

$$-\frac{1}{2}|\underset{n_x}{2}\,\underset{n_y}{0}\,\underset{n_z}{0}\rangle - \frac{1}{2}|020\rangle + |002\rangle$$

You need to normalize them.

Anyway, this last part is just to check if you get the idea