Quiz I, Keys

(1)

(a) You want to calculate $\langle m | a_+ | n \rangle$

use $a_+ | n \rangle = \sqrt{n+1}\, | n+1 \rangle$

$$\langle m | a_+ | n \rangle = \sqrt{n+1}\; \delta_{m,n+1} \qquad m = n+1 \text{ only}$$

nonzero

$$\langle 1 | a_+ | 0 \rangle = 1$$

$$\langle 2 | a_+ | 1 \rangle = \sqrt{2}$$

$$\langle 3 | a_+ | 2 \rangle = \sqrt{3}$$

$$\langle 4 | a_+ | 3 \rangle = \sqrt{4}$$

Thus

$$a_+ = \begin{array}{c c} & \begin{array}{cccc} |0\rangle & |1\rangle & |2\rangle & |3\rangle \end{array} \\ \begin{array}{c} \langle 0| \\ \langle 1| \\ \langle 2| \\ \langle 3| \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \end{array}$$

infinite Matrix

(b) Eq (2.69)

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a_+ + a_-)$$

$$a_- | n \rangle = \sqrt{n}\, | n-1 \rangle$$

$$\langle m | a_- | n \rangle = \sqrt{n}\; \delta_{m,n-1} \qquad m = n-1 \text{ only}$$

$$\langle 0 | a_- | 1 \rangle = 1$$

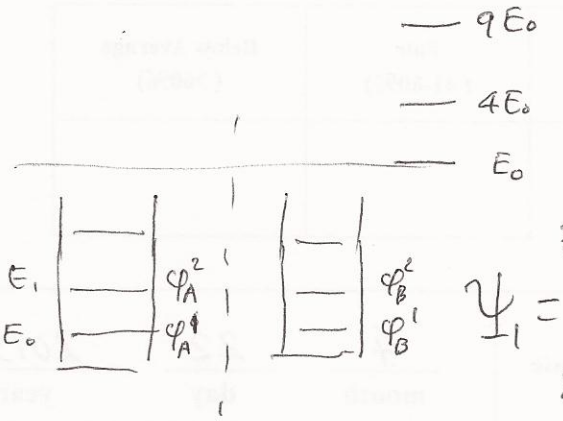
$$\langle 1 | a_- | 2 \rangle = \sqrt{2}$$

$$\langle 2 | a_- | 3 \rangle = \sqrt{3}$$

Combine with part (a)

$$x = \sqrt{\frac{\hbar}{2m\omega}} \begin{array}{c c} & \begin{array}{ccccc} |0\rangle & |1\rangle & |2\rangle & |3\rangle & \end{array} \\ \begin{array}{c} \langle 0| \\ \langle 1| \\ \langle 2| \\ \langle 3| \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & - \\ 1 & 0 & \sqrt{2} & 0 & - \\ 0 & \sqrt{2} & 0 & \sqrt{3} & - \\ 0 & 0 & \sqrt{3} & 0 & - \end{pmatrix} \end{array}$$

(2) For each infinite square well



First wave function

$$\Psi_1 = \frac{1}{\sqrt{2}}\left(\varphi_A^1 + \varphi_B^1\right)$$ no node

2nd wave function

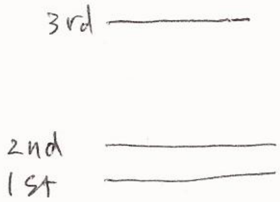
$$\Psi_2 = \frac{1}{\sqrt{2}}\left(\varphi_A^1 - \varphi_B^2\right)$$ one node

small energy differenc

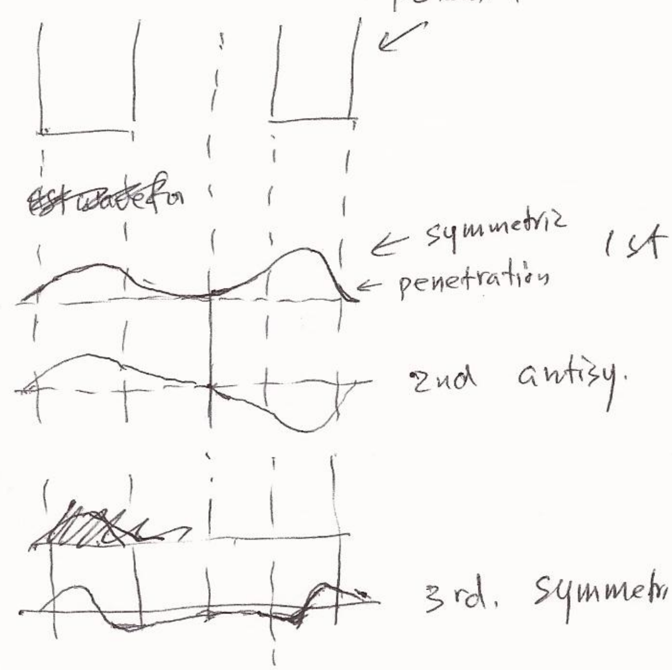
3rd wave function

$$\Psi_3 \quad \frac{1}{\sqrt{2}}\left(\varphi_A^2 + \varphi_B^2\right)$$ two nodes

Energies



wavefunctionr



(3) Use results from your homework

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Let $m=0$ state of S_x be written as $|\chi\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$S_x |\chi\rangle = 0\,|\chi\rangle$$

$$\frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

$$b = 0$$
$$a + c = 0 \quad \Rightarrow \quad a = -c$$
$$b = 0$$

$$|\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

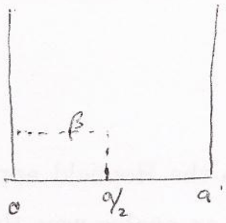
the probability of finding $|u\rangle$ in the state $|\chi\rangle$ is given by the square of $\langle \chi | u \rangle$

$$\langle \chi | u \rangle = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{26}} (1, 0, -1) \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$$

$$= \frac{1}{\sqrt{52}} (1+3) = \frac{4}{\sqrt{52}}$$

$$P = |\langle \chi | u \rangle|^2 = \frac{16}{52} = \frac{4}{13}$$

(4)



at $t=0$

$$\psi_0(x) = \beta \quad 0 \le x < a/2$$
$$= 0$$

Normalize $\int |\psi_0(x)|^2 dx = \beta^2 \cdot \frac{a}{2} = 1$

$$\beta = \sqrt{\frac{2}{a}}$$

(a)
$$\psi_0(x) = \begin{cases} \sqrt{2/a} & 0 \le x < a/2 \\ 0 & a/2 < x < a \end{cases}$$

(b) The eigenstates of an infinite square well

$$\varphi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \qquad E_n = n^2 E_1 \qquad E_1 = \frac{\hbar^2 \pi^2}{2ma^2}$$

The time-dependent wavefunctia

$$\Psi(x,t) = \sum_{n=1}^{\infty} a_n \varphi_n(x) e^{-iE_n t/\hbar}$$

(c)
$$a_n = \langle \varphi_n | \psi_0 \rangle$$

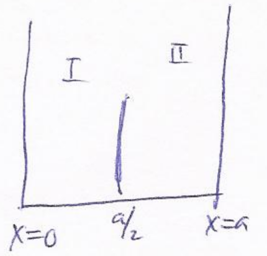
$$a_1 = \int_0^a \varphi_1(x)\, \psi_0(x)\, dx = \int_0^{a/2} \sqrt{\frac{2}{a}} \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}\, dx = \frac{2}{\pi}$$

$$a_2 = \int_0^{a/2} \sqrt{\frac{2}{a}} \sqrt{\frac{2}{a}} \sin \frac{2\pi x}{a}\, dx = -\frac{2}{\pi}$$

prob. for $n=1$ $\quad |a_1|^2 \quad$ indep. of time

$n=2$ $\quad |a_2|^2$;

(5)



$\psi_I = \sin kx$ $\qquad E = \dfrac{\hbar^2 k^2}{2m}$

$\psi_{II} = A \sin kx + B \cos kx$

At $x = a/2$ (1) $\dfrac{d\psi_{II}}{dx} - \dfrac{d\psi_I}{dx} = -\dfrac{2m\alpha}{\hbar^2}\psi(a/2)$

$$kA \cos k\frac{a}{2} - kB \sin \frac{ka}{2} - k\cos\frac{ka}{2} = -\frac{2m\alpha}{\hbar^2}\sin\frac{ka}{2}$$

$$k(A-1)\cos\frac{ka}{2} = \left(kB - \frac{2m\alpha}{\hbar^2}\right)\sin\frac{ka}{2}$$

(2) $\psi_{II} = \psi_I$ at $x = a/2$

$$\sin\frac{ka}{2} = A\sin\frac{ka}{2} + B\cos\frac{ka}{2}$$

(3) $\psi_{II} = 0$ at $x = a$

$$A \sin ka + B\cos ka = 0$$

a mess to solve it

If it is the 2nd state, or odd state,

then $\psi(a/2) = 0$ or $\sin k\frac{a}{2} = 0$

thus $B = 0$

$$k = \frac{2\pi}{a}$$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2\pi^2}{2m} \quad (4)$$

No change