HW 5

1. Griffiths 4.45 p192

From wikipedia, the radius of the proton is $R_p = 0.8775 \times 10^{-15}$ m and the radius of the deuteron is $R_d = 2.1424 \times 10^{-15}$ m.
The Bohr radius $a = 0.529 \times 10^{-10}$ m.
Therefore the size of the hydrogen nucleus is about 4 or 5 orders of magnitude smaller than the expansion of the ground state of hydrogen, which has a size of the Bohr radius.

(a) The ground state wavefunction of H:

$$\Psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

The probability of finding the electron inside a sphere of radius b is

$$\begin{aligned}
P_{r<b} &= \int_0^b dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \; |\Psi_{100}(r,\theta,\phi)|^2 r^2 \sin\theta \\
&= 2\pi \int_0^b dr \int_0^\pi d\theta \; \frac{1}{\pi a^3} e^{-2r/a} r^2 \sin\theta \\
&= \frac{2}{a^3}\left(\int_0^b dr\, r^2 e^{-2r/a}\right)\left(\int_0^\pi \sin\theta\, d\theta\right) \\
&= \frac{2}{a^3}\cdot\left(-\frac{1}{4}\right)a\cdot\left[e^{-\frac{2r}{a}}(a^2+2ar+2r^2)\right]_0^b \cdot 2 \\
&= -\frac{1}{a^2}\left[e^{-\frac{2b}{a}}(a^2+2ab+2b^2) - a^2\right] \\
&= 1 - e^{-\frac{2b}{a}}\left(1+\frac{2b}{a}+\frac{2b^2}{a^2}\right)
\end{aligned}$$

This is the exact answer.

(b). $\epsilon \equiv \frac{2b}{a} \ll 1$

$$
\begin{aligned}
P_{r<b} &= 1 - e^{-\frac{2b}{a}}\left(1 + \frac{2b}{a} + \frac{2b^2}{a^2}\right) \\
&= 1 - e^{-\epsilon}\left(1 + \epsilon + \frac{\epsilon^2}{2}\right) \\
&\cancel{\approx 1 - (1-\epsilon)(1+\epsilon} \\
&\approx 1 - \left(1 - \epsilon + \frac{\epsilon^2}{2} - \frac{\epsilon^3}{6}\right)\left(1 + \epsilon + \frac{\epsilon^2}{2}\right) \\
&= 1 - \left[1 \cancel{-\epsilon} + \frac{\epsilon^2}{2} - \frac{\epsilon^3}{6} \cancel{+\epsilon} - \epsilon^2 + \cancel{\frac{\epsilon^3}{2}} - \frac{\epsilon^4}{6} + \frac{\epsilon^2}{2} - \cancel{\frac{\epsilon^3}{2}} + \frac{\epsilon^4}{4} - \frac{\epsilon^5}{12}\right] \\
&= 1 - \left[1 - \frac{\epsilon^3}{6} + \frac{\epsilon^4}{12} + \dots\right] \\
&= \frac{1}{6}\epsilon^3 + O(\epsilon^4) \\
&\approx \frac{1}{6}\epsilon^3 = \frac{1}{6}\left(\frac{2b}{a}\right)^3 = \frac{4}{3}\left(\frac{b}{a}\right)^3
\end{aligned}
$$

The lowest order is ϵ^3, or $\left(\frac{b}{a}\right)^3$, $P_{r<b} \approx \frac{4}{3}\left(\frac{b}{a}\right)^3$

(c) If we assume that $\psi(r)$ is essentially constant over the tiny sphere with radius b, then $P_{r<b}$ can be obtained as

$$P_{r<b} \approx \frac{4}{3}\pi b^3 \cdot |\psi(r=0)|^2 = \frac{4}{3}\pi b^3 \cdot \frac{1}{\pi a^3} = \frac{4}{3}\left(\frac{b}{a}\right)^3$$

which is the same as the result in (b).

(d) $b \approx 10^{-15}\,m$, $a \approx 0.5 \times 10^{-10}\,m$

$$P_{r<b} \approx \frac{4}{3}\left(\frac{b}{a}\right)^3 = \frac{4}{3} \cdot \left(\frac{10^{-15}}{0.5 \times 10^{-10}}\right)^3 = \frac{32}{3} \times 10^{-15} = 1.07 \times 10^{-14}$$

which is an extremely small probability.

2. First we want to prove that $L_\pm Y_\ell^m = \hbar\sqrt{\ell(\ell+1)-m(m\pm1)}\, Y_\ell^{m\pm1}$

This is also problem 4.18 of Griffiths.

Let us assume $L_+ Y_\ell^m = A Y_\ell^{m+1}$ and $L_- Y_\ell^m = B Y_\ell^{m-1}$

From [4.112] of Griffiths

$$L^2 = L_+L_- + L_z^2 - \hbar L_z = L_-L_+ + L_z^2 + \hbar L_z \qquad (*)$$

Let us first use the latter relation, from which

$$L_-L_+ = L^2 - L_z^2 - \hbar L_z$$

$$\Rightarrow \quad L_-L_+ Y_\ell^m = L^2 Y_\ell^m - L_z^2 Y_\ell^m - \hbar L_z Y_\ell^m = \left[\hbar^2 \ell(\ell+1) - \hbar^2 m^2 - \hbar^2 m\right] Y_\ell^m$$

$$= \hbar^2\left[\ell(\ell+1) - m(m+1)\right] Y_\ell^m$$

Now multiply $(Y_\ell^m)^*$ on the left for both sides and then take an integration over angle

$$\Rightarrow \int (Y_\ell^m)^* L_-L_+ Y_\ell^m \, d\Omega = \hbar^2\left[\ell(\ell+1)-m(m+1)\right] \underbrace{\int (Y_\ell^m)^* Y_\ell^m \, d\Omega}_{=1}$$

use $(AB)^\dagger = B^\dagger A^\dagger$ and $L_+^\dagger = L_-$, $L_-^\dagger = L_+$

$$\Rightarrow \int (L_+ Y_\ell^m)^\dagger L_+ Y_\ell^m \, d\Omega = \hbar^2\left[\ell(\ell+1) - m(m+1)\right]$$

$$\Rightarrow \int (Y_\ell^{m+1})^* A^* A Y_\ell^{m+1} \, d\Omega = \hbar^2\left[\ell(\ell+1) - m(m+1)\right]$$

$$\Rightarrow |A|^2 \underbrace{\int (Y_\ell^{m+1})^* Y_\ell^{m+1} \, d\Omega}_{=1} = \hbar^2\left[\ell(\ell+1) - m(m+1)\right]$$

$$\Rightarrow \quad A = \hbar\sqrt{\ell(\ell+1) - m(m+1)}$$

Similarly, if you use the first relation of $(*)$ you will get

$$B = \hbar\sqrt{\ell(\ell+1) - m(m-1)}$$

$$\begin{cases} L_+ = L_x + iL_y \\ L_- = L_x - iL_y \end{cases} \Rightarrow \begin{cases} L_x = \frac{1}{2}(L_+ + L_-) \\ L_y = \frac{1}{2i}(L_+ - L_-) \end{cases}$$

$$\langle Y_{\ell m_1} | L_x | Y_{\ell m_2} \rangle$$
$$= \frac{1}{2}\langle Y_{\ell m_1} | L_+ | Y_{\ell m_2} \rangle + \frac{1}{2}\langle Y_{\ell m_1} | L_- | Y_{\ell m_2} \rangle$$
$$= \frac{1}{2}\langle Y_{\ell m_1} | \hbar\sqrt{\ell(\ell+1) - m_2(m_2+1)} \, | Y_{\ell, m_2+1} \rangle$$
$$+ \frac{1}{2}\langle Y_{\ell m_1} | \hbar\sqrt{\ell(\ell+1) - m_2(m_2-1)} \, | Y_{\ell, m_2-1} \rangle$$
$$= \frac{\hbar}{2}\left[\delta_{m_1, m_2+1}\sqrt{\ell(\ell+1) - m_2(m_2+1)} + \delta_{m_1, m_2-1}\sqrt{\ell(\ell+1) - m_2(m_2-1)} \right]$$

$$\langle Y_{\ell m_1} | L_y | Y_{\ell m_2} \rangle$$
$$= \frac{1}{2i}\left[\langle Y_{\ell m_1} | L_+ | Y_{\ell m_2} \rangle - \langle Y_{\ell m_1} | L_- | Y_{\ell m_2} \rangle \right]$$
$$= \frac{1}{2i}\left[\hbar\sqrt{\ell(\ell+1) - m_2(m_2+1)} \, \langle Y_{\ell m_1} | Y_{\ell, m_2+1} \rangle \right.$$
$$\left. - \hbar\sqrt{\ell(\ell+1) - m_2(m_2-1)} \, \langle Y_{\ell m_1} | Y_{\ell, m_2-1} \rangle \right]$$
$$= \frac{\hbar}{2i}\left[\delta_{m_1, m_2+1}\sqrt{\ell(\ell+1) - m_2(m_2+1)} - \delta_{m_1, m_2-1}\sqrt{\ell(\ell+1) - m_2(m_2-1)} \right]$$

3. $$H = \frac{L_x^2 + L_y^2}{2I_1} + \frac{L_z^2}{2I_2} = \frac{L^2 - L_z^2}{2I_1} + \frac{L_z^2}{2I_2} = \frac{1}{2I_1} L^2 + \frac{I_1 - I_2}{2I_1 I_2} L_z^2$$

Because $Y_\ell^m(\theta, \phi)$ are eigenstates of L^2 and L_z, they are also the eigenstates of H.

The eigenvalues of H are

$$\frac{1}{2I_1}\hbar^2 \ell(\ell+1) + \frac{I_1 - I_2}{2I_1 I_2}\hbar^2 m^2 \equiv \alpha\, \ell(\ell+1) + \beta m^2$$

where we have set $\alpha \equiv \frac{\hbar^2}{2I_1}$ and $\beta = \frac{I_1 - I_2}{2I_1 I_2}\hbar^2 > 0$ (b/c $I_1 > I_2$)

$\ell = 0, 1, 2, \cdots$ $\qquad m = -\ell, -\ell+1, \cdots, \ell-1, \ell$

Suppose $\beta < \alpha$, $\Rightarrow \frac{I_1 - I_2}{2I_1 I_2}\hbar^2 < \frac{\hbar^2}{2I}$, $\quad I_1 < 2I_2$ (i.e., $I_2 < I_1 < 2I_2$)

then the first few eigen levels are sketched below

$12\alpha + 9\beta$ ———— $(\ell, m) = (3, 3)$ or $(3, -3)$

$12\alpha + 4\beta$ ———— $(\ell, m) = (3, 2)$ or $(3, -2)$

$12\alpha + \beta$ ———— $(\ell, m) = (3, 1)$ or $(3, -1)$

12α ———— $(\ell, m) = (3, 0)$

$6\alpha + 4\beta$ ———— $(\ell, m) = (2, 2)$ or $(2, -2)$

$6\alpha + \beta$ ———— $(\ell, m) = (2, 1)$ or $(2, -1)$

6α ———— $(\ell, m) = (2, 0)$

$2\alpha + \beta$ ———— $(\ell, m) = (1, 1)$ or $(1, -1)$

2α ———— $(\ell, m) = (1, 0)$

0 ———— $(\ell, m) = (0, 0)$

If β is very big, then the ordering of the levels can be complicated and this case will not be considered here.

4. In spherical coordinates
$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

$$\varphi(x,y,z) = C(xy+yz+zx)e^{-\alpha r^2}$$
$$= \cancel{C r^2 e^{-\alpha r^2}}(\sin^2\theta\sin\phi\cos\phi + \sin\theta\cos\theta\sin\phi + \sin\theta\cos\theta\cos\phi)$$

Because angular momentum is determined only by the angular part of the wavefunction, we shall only focus on the angular part: $Y(\theta,\phi) = A(\sin^2\theta\sin\phi\cos\phi + \sin\theta\cos\theta\sin\phi + \sin\theta\cos\theta\cos\phi)$

where A is the normalization constant.

The normalization condition is

$$1 = \int_0^{2\pi}\int_0^{\pi} |Y(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi$$
$$= |A|^2 \int_0^{2\pi}\int_0^{\pi} (\sin^2\theta\sin\phi\cos\phi + \sin\theta\cos\theta\sin\phi + \sin\theta\cos\theta\cos\phi)^2 \sin\theta\, d\theta\, d\phi$$
$$= |A|^2 \cdot \frac{4\pi}{5} \quad \Rightarrow \quad A = \sqrt{\frac{5}{4\pi}}$$

The eigenstates of the angular wavefunction are spherical harmonics. Therefore $Y(\theta,\phi)$ can be written as a linear combination of spherical harmonics

$$Y(\theta,\phi) = \sum_{lm} C_l^m Y_l^m(\theta,\phi)$$

where $C_l^m = \langle Y_l^m | Y\rangle = \int_0^{2\pi}\int_0^{\pi} [Y_l^m]^* Y \sin\theta\, d\theta\, d\phi$

Y_ℓ^m are eigenstates of L^2 and L_z with

$$L^2 Y_\ell^m = \hbar^2 \ell(\ell+1) Y_\ell^m \qquad L_z Y_\ell^m = \hbar m Y_\ell^m$$

The probability that a measurement on L^2 yields 0 (i.e., $l=0, m=0$) is

$$|C_0^0|^2 = |\langle Y_0^0 | Y \rangle|^2 \qquad \boxed{Y_0^0 = \sqrt{\frac{1}{4\pi}}}$$

$$= \left| \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sqrt{\frac{1}{4\pi}} \cdot \sqrt{\frac{5}{4\pi}} \cdot (\sin^2\theta \sin\phi \cos\phi + \sin\theta\cos\theta\sin\phi + \sin\theta\cos\theta\cos\phi)\sin\theta \right|^2$$

$$= 0$$

The probability that a measurement on L^2 yields $6\hbar^2$ (i.e., $l=2$) is

$$|C_2^m|^2 = |C_2^{-2}|^2 + |C_2^{-1}|^2 + |C_2^0|^2 + |C_2^1|^2 + |C_2^2|^2$$

Using Mathematica, I get

$$C_2^{-2} = -\frac{i}{\sqrt{6}} \quad \Rightarrow \quad |C_2^{-2}|^2 = \frac{1}{6}$$

$$C_2^{-1} = \frac{1-i}{\sqrt{6}} \quad \Rightarrow \quad |C_2^{-1}|^2 = \frac{1}{3}$$

$$C_2^{0} = 0 \quad \Rightarrow \quad |C_2^{0}|^2 = 0$$

$$C_2^{1} = -\frac{1+i}{\sqrt{6}} \quad \Rightarrow \quad |C_2^{1}|^2 = \frac{1}{3}$$

$$C_2^{2} = \frac{i}{\sqrt{6}} \quad \Rightarrow \quad |C_2^{2}|^2 = \frac{1}{6}$$

$$\Rightarrow |C_2^m|^2 = \frac{1}{6} + \frac{1}{3} + 0 + \frac{1}{3} + \frac{1}{6} = 1$$

5. (a) $$|jm\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} \underbrace{\langle j_1 j_2; m_1 m_2 | jm\rangle}_{\text{C-G coefficients}} |j_1 m_1\rangle |j_2 m_2\rangle$$

There are a lot of places to find the C-G coefficients. If you know how to read the C-G coefficient table (Table 4.8) on page 188, you can find them there. Besides the traditional C-G tables, I find that there are some on-line C-G calculators which are extremely straightforward to use, for example, the *WolframAlpha* Clebsch-Gordan calculator. You just input the values of j_1, j_2, m_1, m_2, j, m in the corresponding boxes and you get the C-G ~~co~~ coefficient out.

You can check that $\frac{1}{\sqrt{5}}, \sqrt{\frac{3}{5}}, \frac{1}{\sqrt{5}}$ are indeed the correct C-G coefficients:

$$\underset{j\ m}{|3\ 0\rangle} = \frac{1}{\sqrt{5}} \underset{j_1\ m_1}{|2\ 1\rangle} \underset{j_2\ m_2}{|1\ {-1}\rangle} + \sqrt{\frac{3}{5}} \underset{j_1\ m_1}{|2\ 0\rangle} \underset{j_2\ m_2}{|1\ 0\rangle} + \frac{1}{\sqrt{5}} \underset{j_1\ m_1}{|2\ {-1}\rangle} \underset{j_2\ m_2}{|1\ 1\rangle}$$

(b) Strickly speaking, $|20\rangle$ or $|10\rangle$ can be expanded in many different ways depending on the value of j_1 and j_2. Here we follow the way that $|30\rangle$ was expanded, i.e., $j_1=2$, $j_2=1$. Then

$$|2\ 0\rangle = \frac{1}{\sqrt{2}}|2\ 1\rangle|1\ {-1}\rangle + 0\cdot|2\ 0\rangle|1\ 0\rangle - \frac{1}{\sqrt{2}}|2\ {-1}\rangle|1\ 1\rangle$$

$$|1\ 0\rangle = \sqrt{\frac{3}{10}}|2\ 1\rangle|1\ {-1}\rangle - \sqrt{\frac{2}{5}}|2\ 0\rangle|1\ 0\rangle + \sqrt{\frac{3}{10}}|2\ {-1}\rangle|1\ 1\rangle$$

(c) $\langle 1\,0 | 2\,0 \rangle = \left(\sqrt{\tfrac{3}{10}} \langle 2\,1|_1 \langle 1\,-1|_2 - \sqrt{\tfrac{2}{5}} \langle 2\,0|_1 \langle 1\,0|_2 + \sqrt{\tfrac{3}{10}} \langle 2\,-1|_1 \langle 1\,1|_2 \right)$

$\cdot \left(\tfrac{1}{\sqrt{2}} |2\,1\rangle_1 |1\,-1\rangle_2 + 0 \cdot |2\,0\rangle_1 |1\,0\rangle_2 - \tfrac{1}{\sqrt{2}} |2\,-1\rangle_1 |1\,1\rangle_2 \right)$

$= \sqrt{\tfrac{3}{10}} \cdot \tfrac{1}{\sqrt{2}} - \sqrt{\tfrac{3}{10}} \cdot \tfrac{1}{\sqrt{2}} = 0$

$\langle 1\,0 | 3\,0 \rangle = \left(\sqrt{\tfrac{3}{10}} \langle 2\,1| \langle 1\,-1| - \sqrt{\tfrac{2}{5}} \langle 2\,0| \langle 1\,0| + \sqrt{\tfrac{3}{10}} \langle 2\,-1| \langle 1\,1| \right)$

$\cdot \left(\tfrac{1}{\sqrt{5}} |2\,1\rangle |1\,-1\rangle + \sqrt{\tfrac{3}{5}} |2\,0\rangle |1\,0\rangle + \tfrac{1}{\sqrt{5}} |2\,-1\rangle |1\,1\rangle \right)$

$= \sqrt{\tfrac{3}{10}} \cdot \tfrac{1}{\sqrt{5}} - \sqrt{\tfrac{2}{5}} \cdot \sqrt{\tfrac{3}{5}} + \sqrt{\tfrac{3}{10}} \cdot \tfrac{1}{\sqrt{5}} = 0$

$\langle 2\,0 | 3\,0 \rangle = \left(\tfrac{1}{\sqrt{2}} \langle 2\,1| \langle 1\,-1| + 0 \cdot \langle 2\,0| \langle 1\,0| - \tfrac{1}{\sqrt{2}} \langle 2\,-1| \langle 1\,1| \right)$

$\cdot \left(\tfrac{1}{\sqrt{5}} |2\,1\rangle |1\,-1\rangle + \sqrt{\tfrac{3}{5}} |2\,0\rangle |1\,0\rangle + \tfrac{1}{\sqrt{5}} |2\,-1\rangle |1\,1\rangle \right)$

$= \tfrac{1}{\sqrt{2}} \cdot \tfrac{1}{\sqrt{5}} - \tfrac{1}{\sqrt{2}} \tfrac{1}{\sqrt{5}} = 0$

So $|1\,0\rangle$, $|2\,0\rangle$, $|3\,0\rangle$ are orthogonal to each other.

6. 3D isotropic harmonic oscillator.

(a) $V(r) = \frac{1}{2} m\omega^2 r^2$

Because the potential is spherically symmetric, the Schrödinger equation can be handled by separation of variables in spherical coordinates. The wavefunction is $\Psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\, Y_l^m(\theta,\phi)$. where the radial part of the wavefunction $R_{nl}(r)$ satisfies the following equation

$$\left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2}{dr^2} r + \frac{1}{2} m\omega^2 r^2 + \frac{l(l+1)\hbar^2}{2mr^2}\right] R_{nl}(r) = E_{nl} R_{nl}(r)$$

It is quite tricky to work this radial equation out. Here I just write down the solution

$$R_{n_r l} = \sum_{k=0}^{\infty} a_k\, y^{l+2k}\, e^{-y^2/2}$$

where $y = \sqrt{\frac{m\omega}{\hbar}}\, r$

and $E_{n_r l} = \left(2n_r + l + \frac{3}{2}\right)\hbar\omega$

$n_r = 0, 1, 2, \cdots \qquad l = 0, 1, 2, \cdots$

(b) The Schrödinger equation of the 3D isotropic harmonic oscillator in Cartesian coordinates is

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{1}{2} m\omega^2 (x^2+y^2+z^2)\psi = E\psi$$

Let $\psi(x,y,z) = X(x)\, Y(y)\, Z(z)$, plug into the above equation

$$-\frac{\hbar^2}{2m}\left(\frac{d^2X(x)}{dx^2} \cdot YZ + \frac{d^2Y}{dy^2} ZX + \frac{d^2Z}{dz^2} XY\right)$$
$$+ \frac{1}{2} m\omega^2 (x^2+y^2+z^2) XYZ = EXYZ$$

divide XYZ both sides

$$\left(-\frac{\hbar^2}{2m}\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{2}m\omega^2x^2\right)+\left(-\frac{\hbar^2}{2m}\frac{1}{Y}\frac{d^2Y}{dy^2}+\frac{1}{2}m\omega^2y^2\right)$$

$$+\left(-\frac{\hbar^2}{2m}\frac{1}{Z}\frac{d^2Z}{dz^2}+\frac{1}{2}m\omega^2z^2\right)=E$$

The first term is a function only of x, the second term only of y, and the third term only of z. For the above equation to hold, each term should equal a constant:

$$\begin{cases} -\frac{\hbar^2}{2m}\frac{d^2X}{dx^2}+\frac{1}{2}m\omega^2x^2X=E_xX & \leftarrow \text{1D harmonic oscillator} \\ -\frac{\hbar^2}{2m}\frac{d^2Y}{dy^2}+\frac{1}{2}m\omega^2y^2Y=E_yY & \leftarrow \text{1D} \text{ ------} \\ -\frac{\hbar^2}{2m}\frac{d^2Z}{dz^2}+\frac{1}{2}m\omega^2z^2Z=E_zZ & \leftarrow \text{1D} \text{ ------} \end{cases}$$

and $E=E_x+E_y+E_z$

3D harmonic oscillator is decomposed into 3 1D harmonic oscillators.

We know that

$$E_x=(n_x+\tfrac{1}{2})\hbar\omega \qquad n_x=0,1,2,\cdots$$

$$E_y=(n_y+\tfrac{1}{2})\hbar\omega \qquad n_y=0,1,2,\cdots$$

$$E_z=(n_z+\tfrac{1}{2})\hbar\omega \qquad n_z=0,1,2,\cdots$$

$$\Rightarrow E=E_x+E_y+E_z=(n_x+n_y+n_z+\tfrac{3}{2})\hbar\omega$$

$$\equiv(n+\tfrac{3}{2})\hbar\omega \quad \text{with } n=n_x+n_y+n_z$$

(c) The first 4 energy levels are

E	$n_x\ n_y\ n_z$	degeneracy	From spherical coordinates $n_r\ l$	degeneracy $=(2l+1)$
$\frac{3}{2}\hbar\omega$	0 0 0	1	0 0	1
$\frac{5}{2}\hbar\omega$	1 0 0 0 1 0 0 0 1	3	0 1	3
$\frac{7}{2}\hbar\omega$	2 0 0 0 2 0 0 0 2 1 1 0 1 0 1 0 1 1	6	1 0 0 2	1+5=6
$\frac{9}{2}\hbar\omega$	3 0 0 0 3 0 0 0 3 2 1 0 2 0 1 1 2 0 0 2 1 1 0 2 0 1 2 1 1 1	10	1 1 0 3	3+7=10