AQM-2013f HW4

1. Calculate the energy levels of a 3D infinite spherical square well potential with the radius r=a. Order the energies for the first ten levels, write down their wavefunctions and indicate the degeneracy of each level.

2. If the 3D infinite spherical square well is located between the two radii at r=a and r=b (b>a), show the steps you need to take in order to calculate the binding energies. [No solution is requested].

3. You have solved the eigenvalues of the infinite spherical well problem in problem 1.

   (a) Write down the normalized radial wave functions for n=1, 2 , and l=0, 1, 2. Do the integration numerically using any computer packages you want.
   (b) Plot the wavefunctions for l=0, n=1 and n=2, together on the same graph, also show that the two radial wavefunctions are orthogonal to each other.
   (c) Plot the wavefunctions for n=1, l=0, 1, 2. Are they orthogonal to each other? If they are not, does it mean that we are using non- orthogonal basis functions?
   Make sure that you label your figures (even by hand is ok).

4. To get you more comfortable with spherical Bessel function $j_l(x)$ and spherical Neumann function $n_l(x)$, do the following:

   (a) Write down the asymptotic form of $j_l(x)$ and $n_l(x)$ for $x \rightarrow 0$. You can find them in your textbook or from the web.
   (b) Do the same for the limit of for $x \rightarrow \infty$.
   (c) On page 401 of your book, you will find reference to the so-called spherical Hankel functions, see eq. [11.19]. Use the results of (b) to show that the asymptotic form for $h_l^{(1,2)}(x)$ is indeed given by the expression in Table 11.1.
   (d) If you set x to ix in (c), you find that you get an exponential increasing and decreasing functions, respectively. Show which one is increasing and which one is decreasing.

5. This exercise will allow you to use what you have learned to do many types of "hydrogen atom" in physics. Recall that the standard hydrogen atom has energy levels given by $E_n = -13.6\ eV/n^2$ where the nuclear charge is one and the mass is that of the electron's.

   (a) Calculate the difference of the ground state (n=1) energies of a hydrogen atom and of a deuterium atom, express the answers in meV (10^{-3} eV).

(b) A positronium is made of (e^+e^-), i.e., of a positron and an electron. Calculate its ground state energy.

(c) A muonic hydrogen is made of ($\mu^- p^+$), where the mass of the muon is 206 times of that of an electron, calculate the ground state energy of such an 'atom". What is the transition energy between the n=2 and n=1 levels? Express it in eV and in wavelength of the radiation emitted.

(d) If you replace the charge of the proton in atomic hydrogen by a nuclear charge of Z, what will be the expression for the energy levels, i.e., how it scales with the charge Z? (Hint: Start with the radial equation for charge Z, define s=Zr where s is the new scaled distance. Rewrite the radial equation in terms of s, and you should be able to show that it scales like Z^2. No need to solve the equation again.)

(e) In a semiconductor, one can have so-called excitons. They can be considered as a positively charged "hole", with mass, say 0.5 m_e and a negatively charged " electron" with mass, say 0.5 m_e (they are not really electrons and holes, but are called quasi-particles). They interact in a dielectric medium where the Coulomb interaction is reduced by a factor κ. Let us take κ=16, calculate the binding energy of the exciton, expressed in meV.

[Questions from DE problem set]

6. Consider a particle in an infinitely deep three-dimensional box with the length on each side equal to
a. Write down the Hamiltonian describing such a particle.
b. Find the first 5 lowest eigenenergies.
c. Find the degeneracy of each energy eigenstate above.
d. If there are 10 noninteracting, indistinguishable fermions in such a box, what is the ground state energy of the whole system? Consider separately the cases for spin 1/2 and spin 3/2 particles.

[This is from DE problem list. You cannot do part d yet. Will consider it later]

7. Go to webpage
http://www.phys.ksu.edu/graduate/current/departmental-exams/QM_student.pdf

do problem 14. This is similar to the question 3 in exam 1. Prove that you can do it with ease now.

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