AQM-2013f HW1

You are allowed and encourged to do integrals using MathLab or Mathematica.

1. Do problem 1.5 on page 14 of Griffiths.

2. (a) write down the Hamiltonian H of a one-dimensional harmonic oscillator.

(b) The ground state wavefunction has the form

$$\varphi_0(x) = A \exp \left[\frac{-m\omega x^2}{2\hbar}\right]$$

Find the normalization constant A.

(c) The first excited state has the functional form

$$\varphi_1(x) = (B + C x) \exp \left[\frac{-m\omega x^2}{2\hbar}\right]$$

Use the orthogonality and normalization conditions to find B and C.

If you can use software to the integrals, then use the same procedure to find the next state

$$\varphi_2(x) = (D + E x + F x^2) \exp \left[\frac{-m\omega x^2}{2\hbar}\right].$$

(d) Find the energy E_1

$$H \varphi_1(x) = E_1 \varphi_1(x)$$

3. For a one-dimensional harmonic oscillator, define

$$a_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}} (\mp ip + m\omega x),$$

Use the communtation relation $[x,p] = i\hbar$ to prove the following relations

$$[a_-, a_+] = 1$$

$$H = \hbar\omega \left(a_+ a_- + \tfrac{1}{2}\right)$$

$$a_+ \varphi_n = \sqrt{n+1} \quad \varphi_{n+1}$$

$$a_- \varphi_n = \sqrt{n} \quad \varphi_{n-1}$$

$$H \varphi_n = (n+1/2) \hbar\omega$$

4. Check the uncertain relationship

$$\sigma_x \sigma_p \;\; \geq \hbar/2$$

For the ground state and 1st excited state of a one-dimensional harmonic oscillator.

5. Let φ_1(x) and φ_2(x) be the ground and first excited states of a one-dimensional (1D) infinite square-well potential within x=[0,a].

   (a) If at t=0 the wavefunction is given by

   $$\psi_A(x, t = 0) = \frac{1}{\sqrt{2}}\varphi_1(\mathrm{x}) \;+\; \frac{1}{\sqrt{2}}\varphi_2(\mathrm{x})$$

   What is the probability of finding a particle of mass m in the ground state?

   (b) What is the wavefunction $\psi_A(x, t)$ for later time?

   (c) At what later time t_1 will $\psi_A(x, t)$ have the same density distribution?

   (d) Make ten graphs of the density $|\psi_A(x, t)|^2$ for ten time steps from 0 to the smallest t_1.

   (e) If at t=0 the wavefunction is given by

   $$\psi_B(x, t = 0) = \frac{1}{\sqrt{2}}\varphi_1(\mathrm{x}) \;-\!-\; \frac{1}{\sqrt{2}}\varphi_2(\mathrm{x})$$

   Answer questions b and c again.

   (f) Can you think of a way to determine that your initial state is ψ_A or ψ_B ?