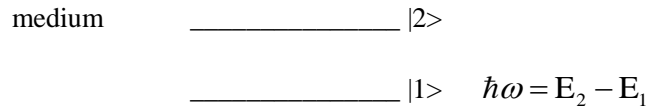


### 3.1. Laser basics --The material is taken from BJ, Chap. 15

#### Preliminary: Einstein A and B coefficients

Consider a two-level system in a radiation field



The number of transitions per unit time from |2> to |1> and from |1> to |2> are given by

$$\frac{dN_2}{dt} = B_{21}\rho(\omega)N_1 = W_{21} N_1$$

$$\frac{dN_1}{dt} = B_{12}\rho(\omega)N_2 + AN_2$$

Where  $W$  is the transition rate,  $A$  is the spontaneous emission rate, and the  $B$  terms give the absorption or stimulated emission. At thermal equilibrium the two equations above are equal, while  $\rho(\omega)$  is given by the Planck distribution law. Since  $I = c\rho$ , we have  $W = B\rho$ . Recall that the absorption cross section  $\sigma$  is defined by

$$\sigma = \hbar\omega W_{12}/I,$$

i.e., the ratio of energy absorbed per atom per unit intensity. Einstein has shown that  $B_{12} = B_{21}$  and that  $A$  is related to  $B$ , see BJ, Eq. (4.108b).

#### 1. Masers and Lasers

Consider an EM wave of intensity  $I$ , frequency  $\omega$ , passing thru a medium, let

$\frac{d\rho_a}{dt}$  : change of average energy density due to absorption

$\frac{d\rho_s}{dt}$  : change of average energy density due to stimulated emission

$$\frac{d\rho_a}{dt} = -N_1 \hbar\omega W_{21}$$

$$\frac{d\rho_s}{dt} = N_2 \hbar\omega W_{12}$$

$$W_{12} = W_{21}$$

( $N$  is large so spontaneous emission is neglected.) The rate of change of total photon energy in the medium is

$$\frac{d\rho}{dt} = (N_2 - N_1) W_{12} \hbar\omega = \sigma I (N_2 - N_1)$$

If the beam has cross section  $A$  along the  $z$ -axis, then this equation becomes

$$\frac{dI}{dz} = \sigma I (N_2 - N_1)$$

the lasing condition

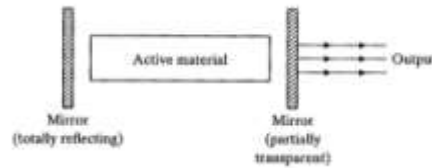
This shows that will intensity will grow exponentially with z when population inversion is achieved— such a medium is called an active medium.

Maser (1954): Townes et al.

Lasers (1958): Schawlow and Townes

Amplifiers used: (maser) → use resonant cavity

laser → active materials between two mirrors



Laser properties:

monochromatic

directional

bright

spatial coherence → between two points  $P_1$  and  $P_2$  → coherent length → coherent area

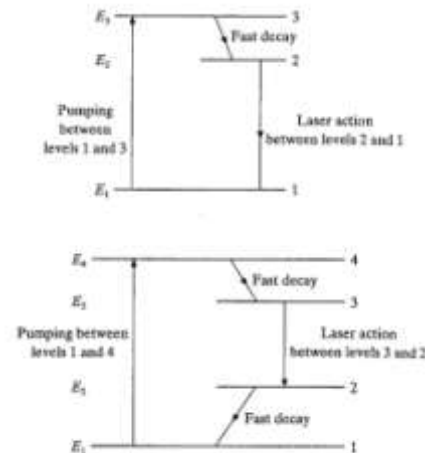
temporal coherence → between  $\tau$  at point P → coherence time

## 2. Methods of Achieving Population Inversion

(a) Amonia maser ⇒ select quantum states by inhomogenous electric field (Townes)

(b) Hydrogen maser ⇒ by magnetic field using inhomogeneous magnetic field (Ramsey)

## 3. Lasers—of different kinds (mostly 3-level or 4-level systems)



(a) The **ruby laser** (A 3-level system) Red~ **6943Å**  
1960 Maiman  $\text{Al}_2\text{O}_3$   $\text{Al}^{3+}$  ions replaced by  $\text{Cr}^{3+}$

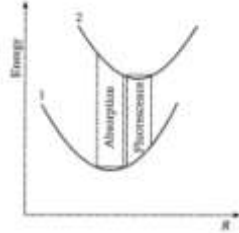
(b) **YAG lasers** (4-level system)  $\text{Yb}^{3+}$  replaced by  $\text{Nd}^{3+}$   $\text{YbAl}_5\text{O}_{12}$  ← host  
Nd:YAG pump 730 nm and 800nm

transition  $\lambda = 1064\text{nm}$

- (c) **Nd:glass**  
 Replace Yb by phosphate or silicate glasses

$\lambda \sim 1054\text{nm}$  (NOVA)

- (d) Ti:sapphire (doping  $\text{Ti}_2\text{O}_3$  into  $\text{Al}_2\text{O}_3$ )



- (e) Dye lasers  
 → gives tunable sources of laser radiation
- (f) Semiconductor lasers  
 Normally use hole-electron recombination II-IV or III-V materials normally weaker power.
- (g) Gas lasers (discharge to pump)

Ne-laser

$5S \rightarrow 3p$  (red,  $6330\text{\AA}$ ) – scanning, low power

$4S \rightarrow 3P$

$5S \rightarrow 4p$

Ar laser ( $\text{Ar}^+$  is the medium)

visible region

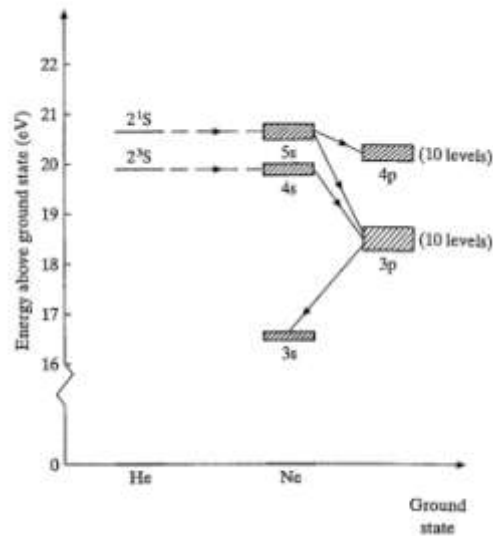
Kr  $e^- + \text{Ar} \rightarrow \text{Ar}^+$

$e^- + \text{Ar}^+(3p^{-1}) \rightarrow \text{Ar}^+(3p^4 4s)$  – short

$4p \rightarrow 4s$  transition

$4p$  – long lifetime

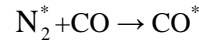
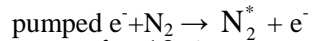
$\text{He}^* + \text{Ne} \rightarrow \text{He} + \text{Ne}^*$



used as a pump for solid state lasers

(h) **CO<sub>2</sub> lasers**

-- transitions involving vibrational levels of CO<sub>2</sub>



$$\lambda = 10.6 \mu\text{m}$$

very powerful, reaches 100kW

(i) excimer lasers (UV regime)

KrF at 249 nm

XeCl 308nm

ArF 193nm

}

very powerful, not narrow

(j) Free-electron lasers - to reach intense short-wavelength light sources

Relativistic e<sup>-</sup> beam in undulators emitted light

scattering by e<sup>-</sup> beam : ranges from infrared to new push toward x-rays

**Flash** (since 2005) : wavelength : from 44 nm to 4.1 nm

Single pulse energy: 10-100 μJ

Pulse duration : 10-70 fs

**LCLS**(since 2010) : photon energy: 0.5 to 10 keV

Pulse energy: 3 mJ at 8.3 keV and 0.83 keV (10<sup>12</sup> photons/pulse)

Pulse duration: tens of fs, but hard to know, chaotic

**Water window:** Between **2.3nm and 4.4 nm**. Water is transparent in this region.

Possible to investigate samples in aqueous solution. For biological samples, carbon atoms are higher opaque but the surrounding water is transparent.

(k) **High-intensity lasers/short pulses**

oscillator → trains of pulses of short duration  
 amplifier

CPA → (1985) Mourou and Strickland

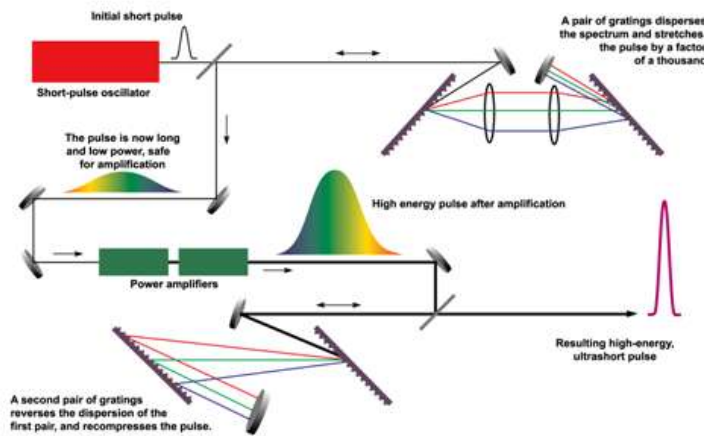
stretch in time by grating ⇒

1990's Nd:YAG  $\lambda = 1064\text{nm}$ ,  $\text{ps} \Rightarrow 10^{12}\text{W} \Rightarrow 10^{18}\text{W/cm}^2$   
 repetition rate 1/shot/min.

Ti: Sapphire--

stretch-- amplify-- compress

[http://en.wikipedia.org/wiki/Chirped\\_pulse\\_amplification](http://en.wikipedia.org/wiki/Chirped_pulse_amplification)



**4. High-resolution spectroscopy**

4.1. **Doppler broadening**

$\omega$ : lab frame,

$$\omega' = \omega - kv = \omega(1 - v/c)$$

$\omega'$ : moving frame

$$k = \omega/c = 2\pi/\lambda$$

$$\Rightarrow \omega' = \omega - kv \xrightarrow{v} \leftarrow \omega'' = \omega + kv$$

Let  $\omega_0$  be the rest frame frequency of the atom,  $\omega' = \omega_0$ ,

Atoms moving with  $v$  absorbs  $\omega$  when  $\omega = \omega_0 + \delta = \omega_0 + kv$  or  $\frac{\delta}{\omega_0} = \frac{v}{c}$ .

Atoms at temperature  $T$  follows Maxwell distribution

$$f(v)dv = \sqrt{\frac{M}{\pi 2k_B T}} e^{-\frac{Mv^2}{2k_B T}} dv = \frac{1}{u\sqrt{\pi}} e^{-v^2/u^2} dv$$

where  $u = \sqrt{2k_B T / M}$  is the most probable speed at T

The absorption has the shape

$$g_D(\omega) = \frac{c}{u\omega_0\sqrt{\pi}} e^{-\frac{v^2}{u^2}(\omega-\omega_0)^2} \leftarrow \text{normalized.}$$

The FWHM  $\frac{\Delta\omega_D}{\omega_0} = 2\sqrt{2\ln 2} \frac{u}{c} \approx 1.7 \frac{u}{c}$

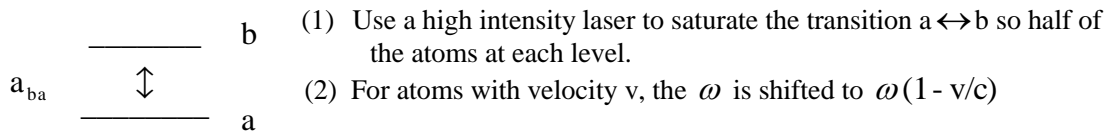
Example: T = 300°K  
 H  $\Delta\omega_D / \omega_0 = 10^{-5}$   
 Cs  $10^{-6}$

Thus Doppler broadening limits resolution to  $10^{-5}$  or  $10^{-6}$ .

#### 4.2. Beating Doppler broadening: The Cross Beam Method

use supersonic jets      small vertical velocity  $\Rightarrow$  lower T  
 needs high-intensity and narrow-freq. bandwidth of laser light.

#### 4.3 Saturation absorption spectroscopy (see BJ p. 843). First used in 1971

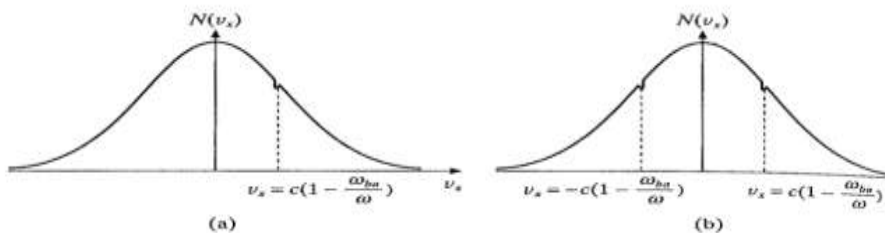


Transition can occur only if  $\omega(1 - v/c) < \Gamma_b / \hbar$        $\Gamma_b$ : natural width

In the intense laser, atoms with velocity in the range above will be depleted to level b. This causes a depletion in the thermal velocity distribution

$$\Delta v = \frac{c\Gamma_b}{\hbar\omega}$$

When a probe beam is used to scan the resonance, the hole will be observed.



4.4. Doppler-free two-photon transitions (two counter propagating beams)

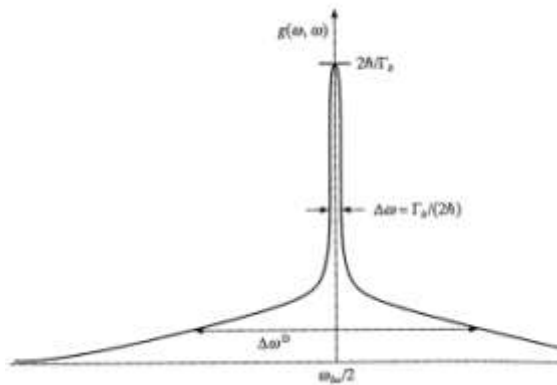
\_\_\_\_\_ (b)  
 \_\_\_\_\_ (a)

If the atom absorbs one photon from each beam, then the Doppler Shift cancels out

$$\omega(1 + v/c) + \omega(1 - v/c) = 2\omega = \omega_{ba}$$

$$\omega = \frac{1}{2}\omega_{ba}$$

Typical two-photon absorption profile from two counter-propagating lasers. The width of the center peak measures the natural width.



Note: All atoms participate in the Doppler-free two-photon absorption. For saturation spectroscopy, only the atoms with specific velocity are absorbing the photons  $\Rightarrow$  measures  $\Gamma_b / 2\hbar$ , i.e., half of the natural width.

4.5. Two-photon spectroscopy of  $1s \rightarrow 2s$  transition in atomic hydrogen

$$\Gamma_b(2s) = 1 \text{ Hz} \quad \text{lifetime} = \frac{1}{8} \text{ sec} \quad \text{wavelength} = 1216 \text{ \AA}$$

Need UV radiation  $\lambda \sim 243 \text{ nm}$  (by frequency doubling)

source of errors:

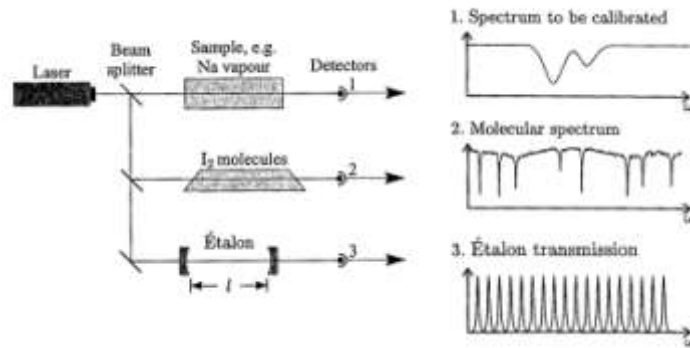
- transition time
- collision broadening
- laser bandwidth (10ns  $\Rightarrow$  100 MHz)
- second order Doppler effect
- A.C. Stark shift

Lots of experiments continuing to achieve ever higher precision. Also works in  $\text{He}^+$  ions.

5. Calibration in laser spectroscopy--how to know the wavelength precisely

Use the frequency of Cs as standard and define the speed of light to define the length.

- (1) Calibration of relative frequency.



(2) Absolute calibration

often takes many steps - the old ways  
 $\Rightarrow$  needs to go to the Cs frequency standard and use chains of lasers to go from microwave to optical

$\rightarrow$  **Frequency comb**—new way—revolutionize high-precision spectroscopy, atomic clocks, in combination with cold atoms.

See: Condiff and Jun Ye, RMP 75, 325 (2003)

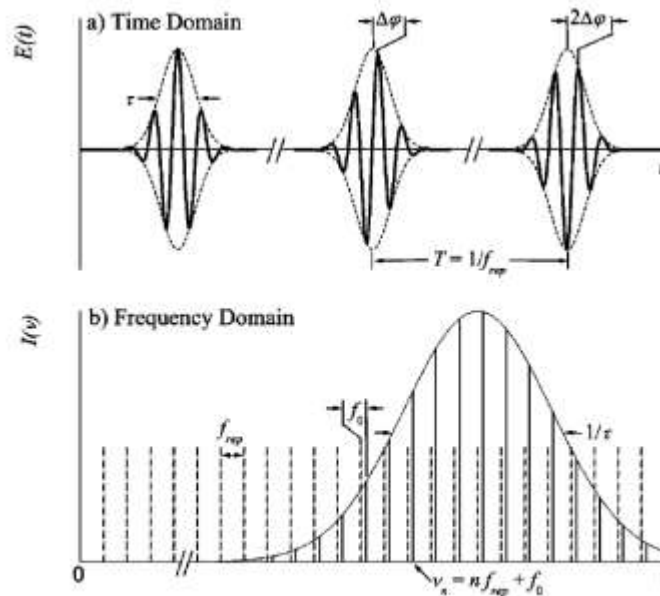


FIG. 5. Summary of the time-frequency correspondence for a pulse train with evolving phase.

The optical frequencies can be expressed as

$$f_n = n f_{rep} + f_0$$

For a broad-band spectra, one can use the second harmonic to beat against the pulse itself, the difference frequency gives

$$2f_n - f_{2n} = 2(nf_{rep} + f_0) - (2nf_{rep} + f_0) = f_0$$

This allows the determination of the frequency offset  $f_0$ .