2.3. Wave packets

When an atom is perturbed by a finite (in time) external field, at the end of the perturbation, the atom is left in a state described by

\[ \Psi(x, t = 0) = \sum_i c_i \varphi_i(x) \]  

(1)

Where we have defined \( t=0 \) when the pulse is over. The coefficients \( c_i \) gives the probability amplitude of finding the atom in an eigenstate \( \varphi_i \). You learn this in quantum mechanics. There you also learn that the wavefunction of the atom after the pulse is over is described by

\[ \Psi(x, t) = \sum_i^{M} c_i \varphi_i(x) e^{-iE_i t} \]  

(2)

Where \( E_i \) is the eigen-energy of the state \( i \). But how do you actually do the measurement? To measure an eigenstate \( |i> \), you have to project out the time-dependent wavefunction over the time interval \( T \) of measurement, i.e., you evaluate

\[ \int_{t+T}^{t+T} <\varphi_n|e^{-iE_nt}\Psi(x, t)> dt = \sum_i^{M} c_i \int_{t+T}^{t+T} <n|i> \exp{i(E_n - E_i)t} dt \]  

(3)

Where the measurements takes from \( t' \) to \( t'+T \), i.e., \( T \) is the measurement time. Of course if the measurement time is finite, the state you project to is not exactly an eigenstate. Anyway, if \( T \) is large, then integral over time on the right-hand side will vanish except for \( i=n \), thus leaving the coefficient \( c_i \) which is the transition amplitude for the atom being in state \( |n> \). In other words, it will give the excitation probability \( |c_i|^2 \).

The situation will be quite different if we are able to make a measurement in a short time. In fact, at the end of an interaction, the electron is left in a wave packet expressed in (2). The expression (2) is mathematically correct, but it gives little information on the nature of this wave packet. To understand wave packets, we will do some exercises.

(a) If the wave packet is made of a few discrete states, say two only, then the modulus square of (2) clearly contains an oscillatory component given by

\[ 2 \text{Re}\{\varphi_1^*\varphi_2 \exp{i(E_1 - E_2)t}\} \]

i.e., the wavefunction will flop with a **beat frequency** which is given by the energy difference between the two levels. If there are many "levels" in the wave packet, then there will be beat frequency for each pair of levels.

(b) If the wave packet consists of a band of continuum states, then depending on the nature of the coefficients, the wave will be a travelling wave packet. This is best illustrated in the powerpoint file shown below.
Comments: The states we discussed here are called **coherent states**, or more precisely, **pure states**. It means that the ensemble of atoms that we are studying are all the same. Even if the system were prepared in pure states, as it evolves, the atoms may interact with other atoms or the environment such that the ensemble becomes a system of mixed states. In such case, we need to formulate the problem using **density matrix** concept.

An analogous situation occurs also for EM waves. A laser light is not always completely coherent. It may be only partially coherent. The latter calls for statistical treatment too.