Homework 4 (due 9/18)

4.1. Exercise 1. (from lecture notes)

4.2. Exercise 2 #11

4.3 You may want to consult Merzbacher P430

Consider $\vec{J} = \vec{\ell} + \vec{s}$

where $\vec{s}$ is spin 1/2.

The eigenstates of J^2 and J_z are given by $\mathcal{Y}_\ell^{jm} = \mathcal{Y}_\ell^{\ell \pm 1/2, m}$

(a) Use CG coeffs in (19.63) to show ($\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$)

$$\mathcal{Y}_\ell^{\ell \pm 1/2, m} = \frac{1}{\sqrt{2\ell+1}} \left(\pm \sqrt{\ell \pm m + 1/2} \right) \alpha \mathcal{Y}_\ell^{m-1/2} + \frac{1}{\sqrt{2\ell+1}} \sqrt{\ell \mp m + 1/2}\ \beta\, \mathcal{Y}_\ell^{m+1/2} \qquad (19.64)$$

(b) Show that $J^2\, \mathcal{Y}_\ell^{jm} = j(j+1)\hbar^2\, \mathcal{Y}_\ell^{jm}$

use $J^2 = \ell^2 + s^2 + 2\vec{\ell} \cdot \vec{s}$

$= \ell^2 + s^2 + \ell_+ s_- + \ell_- s_+ + 2\ell_0 s_0$