homework set-3 (due 9/11)

1. (a) In Q.M, a vector operator satisfies [Le Bellac, p.233]

$$[J_i, V_j] = i\hbar \sum_k \epsilon_{ijk} V_k$$

show that this equation can be rewritten as

$$[\vec{V}, \hat{n}\cdot\vec{L}] = i\hbar\, \hat{n}\times\vec{V}$$

for rotation about $\hat{n}$ in real space.

(b) Let $\vec{V} = \vec{r}$, $\vec{p}$ and $\vec{L}$, respectively. From the eq. above, show that

$$[L_x, y] = i\hbar z$$
$$[L_x, P_y] = i\hbar P_z$$
$$[L_x, L_y] = i\hbar L_z$$

2. In classical physics, $\vec{r}\cdot\vec{L} = 0$ and $\vec{p}\cdot\vec{L} = 0$

Show that in quantum mechanics, both are null operators.

3. Show that

$$\delta(\vec{r}-\vec{r}') = \frac{\delta(r-r')}{r^2} \sum_{\ell,m} Y^*_{\ell m}(\theta,\phi)\, Y_{\ell m}(\theta',\phi')$$

is correct by proving

$$\int f(\vec{r}')\, \delta(\vec{r}-\vec{r}')\, d\vec{r}' = f(\vec{r})$$

4. A classical particle in a central potential is given by

$$H = \frac{P_r^2}{2m} + \frac{L^2}{2mr^2} + V(r)$$

prove that

when $P_r = \frac{1}{r}(\vec{r}\cdot\vec{p})$ is the radial momentum. In Q.M. P_r should be defined by

$$P_r = \frac{1}{2}\left[\frac{1}{r}(\vec{r}\cdot\vec{p}) + (\vec{p}\cdot\vec{r})\frac{1}{r}\right]$$

and $P_r = \frac{\hbar}{i}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$ is the correct differential operator

(Mer p255)