Problem set -7, due Tuesday, Oct 21, 2003

25. A displacement of an object by a vector $\rho$ is represented by a unitary operator $U_r(\rho)$. We have said in the class that this gives

$$U_r^+(\rho)\hat{r}U_r(\rho) = \hat{r} + \rho$$

Show this relation mathematically. Just consider the displacement is in the x-direction. Hint: Show $\hat{r}U_r(\rho) = U_r(\rho)(\hat{r} + \rho)$ instead of the expression above.

26. This is a very simple exercise, but just want to make sure that you know how to do the simple problems. For any angular momentum, in the representation where $\hat{J}^2$ and $\hat{J}_z$ are diagonal, what are the matrix representation of $\hat{J}_x$ and $\hat{J}_y$? Work out the explicit matrices for $J=1/2, 1, 3/2$ and compare your results, for example, (27.26) of Schiff. Note that in this representation the matrices $\hat{J}_x$ and $\hat{J}_z$ are real while $\hat{J}_y$ is purely imaginary.